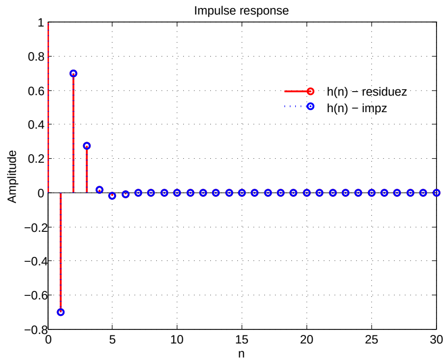

Figure 4.11: Unit impulse response

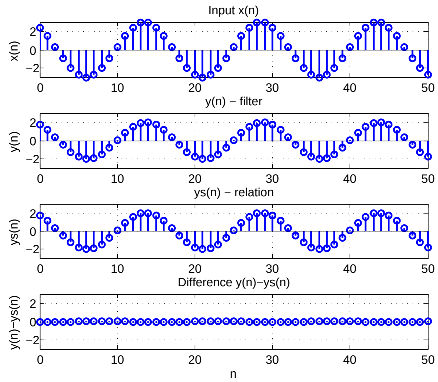

Figure 4.12: Response to a sinusoidal excitation

After running the MATLAB script `L4_6.m` the graphics from Fig. 4.12 are obtained. What do you observe? Is $y(n)$ a sinusoid or not? Explain.

```
% L4_6 - response of an LTIS to a sinusoid
b = [0.3 0.5 0.3]; a = [1 0 0.7]; n = 0:150;
fi = pi/3; A = 3; f = 1/15; N = 15;
x = A*cos(2*pi*f*n+fi); y = filter(b, a, x, [0 0]);
[H, w] = freqz(b, a, 2048); % frequency response function
ind = find(w <= 2*pi*f); we = ind(length(ind));
Hwm = abs(H(we)); Hwp = angle(H(we));
% output seq. obtained using the given relation
ys = A*Hwm*cos(2*pi*f*n + fi + Hwp); n1 = 0:(length(n)-5*N);
subplot(411); stem(n1, x(N5*:length(n)));
title('Input x(n)'); ylabel('x(n)');
subplot(412); stem(n1, y(5*N:length(n)));
title('y(n) - filter'); ylabel('y(n)');
subplot(413); stem(n1, ys(5*N:length(n)));
title('ys(n) - relation'); ylabel('ys(n)');
subplot(414); stem(n1, ys(5*N:length(n)) - y(5*N:length(n)));
title('Difference y(n)-ys(n)'); ylabel('y(n)-ys(n)');
```

4.4 Exercises

1. Demonstrate through a MATLAB scrip, similar with `L4_1.m`, the non-linearity of the system $H\{x(n)\} = x^2(n)$. Consider $a = 3$, $b = -3$, $x_1(n) = \sin(2\pi 0.1n)$ and $x_2(n) = \sin(2\pi 0.15n)$.

2. Find the impulse response of the system described by:

$$H(z) = \frac{1 + z^{-1} + z^{-2} + z^{-3}}{1 - 0.5z^{-1} - 4z^{-2} + 2z^{-3}}.$$

3. Find the impulse response of the system: $H(z) = \dfrac{0.5z^2 + 0.5z}{z^2 - z - 0.5}$.
4. Evaluate the first 100 samples of the impulse response of the system: $H(z) = \dfrac{z}{z-1}$.
5. Evaluate the first 50 samples of the impulse response sequence of the system: $H(z) = \dfrac{z^2+1}{z^3 - 1.9z^2 + 1.55z - 0.425}$.
6. A discrete LTIS is characterized by the constant-coefficient difference equation:
$$y(n) - 1.5\cos\frac{\pi}{8} y(n-1) + 0.95y(n-2) = x(n) + 0.4x(n-1).$$
   - Determine the poles of the system using the `roots` command. If these roots are complex-conjugate, the response of the system contains harmonic components. Represent the real and the imaginary part of the complex sequences: $p_k^n u(n),\ n = \overline{0,30}$, where p_k are the system's poles;
   - Accordingly to the difference equation, the unit impulse response sequence is: $h(n) = (a_1 p_1^n + a_2 p_2^n)\, u(n)$. Determine the constants a_1 and a_2. Evaluate the impulse response using the MATLAB command `impz` (see Example `L4_5.m`);
   - Determine the steady-state response of the system to the complex exponential input sequence (see Example `L4_6.m`) $x(n) = e^{j\omega_0 n}$, $n = \overline{0,60}$, $\omega_0 = \pi/6$.
7. An LTIS is characterized by: $H(z) = \dfrac{(z+0.2)\left(z^2+5\right)}{(z-0.7)\left(z^2 - z + 0.49\right)}$.
   - Represent the poles and the zeros in the z-plane;
   - Evaluate and plot the phase response characteristic. Is this a linear-phase system?
8. Analyze the effect of poles and zeros of a system function $H(z)$ on the magnitude of the frequency response function $|H(\omega)|$, for the systems:
   - $H_1(z) = \left(1 - z_1 z^{-1}\right)\left(1 - z_2 z^{-1}\right)$ where:

     (a) $z_{1,2} = 1$; (b) $z_{1,2} = e^{\pm j\frac{\pi}{6}}$; (c) $z_{1,2} = e^{\pm j\frac{\pi}{3}}$; (d) $z_{1,2} = e^{\pm j\frac{\pi}{2}}$; (e) $z_{1,2} = e^{\pm j\frac{2\pi}{3}}$; (f) $z_{1,2} = e^{\pm j\frac{5\pi}{6}}$; (g) $z_{1,2} = -1$;

     Analyze how $|H(\omega)|$ is modifying accordingly to the zeros position, and represent the zeros in z-plane. What do you observe? Comment on the results.

- $H_2(z) = \dfrac{0.3}{\left(1 - p_1 z^{-1}\right)\left(1 - p_2 z^{-1}\right)}$ where:

  (a) $p_{1,2} = 0.3$; (b) $p_{1,2} = e^{\pm j\frac{\pi}{4}}$; (c) $p_{1,2} = e^{\pm j\frac{\pi}{2}}$; (d) $p_{1,2} = -0.3$.

  Analyze how $|H(\omega)|$ is modifying accordingly to the poles position, and represent the poles in z-plane. What do you observe? Comment on the results.

9. Next transfer functions of some discrete LTIS are considered:

$$H_1(z) = 1 - 4z^{-1} + 4z^{-2}; \qquad H_4(z) = \frac{\left(1 + z^{-1}\right)^2}{1 - z^{-1} + 0.25z^{-2}};$$

$$H_2(z) = 1 + 4z^{-1} + 4z^{-2}; \qquad H_5(z) = \frac{\left(1 - z^{-1}\right)^2}{1 - z^{-1} + 0.25z^{-2}};$$

$$H_3(z) = 1 - z^{-1} + 0.25z^{-2}; \qquad H_6(z) = \frac{1}{1 - z^{-1} + 0.25z^{-2}};$$

- Plot the pole-zero diagrams for the considered systems;
- Represent the frequency response characteristics for each system. Specify what type of system is described by each transfer function;
- Evaluate and plot the unit impulse and the unit step response for all systems.

10. Two causal systems are considered. Determine which one is stable. Comment your answer.

$$H_1(z) = \frac{1 - 0.6z^{-1} + 1.15z^{-2} - 0.98z^{-3} + 0.98z^{-4}}{1 + 1.27z^{-1} + 2.02z^{-2} + 1.54z^{-3} + 0.98z^{-4}};$$

$$H_2(z) = \frac{2 - 2.54z^{-1} + 5z^{-2} - 4.3z^{-3} + 3.27z^{-4}}{1 - 0.77z^{-1} + 0.82z^{-2} + 0.41z^{-3} + 0.51z^{-4}}.$$

11. For the sequence: $x(n) = (0.9)^n \sin(0.2n)$, $n = \overline{0, 99}$, find the impulse response after you evaluate $X(z)$.

Hint: The z-transform of the sequence $x(n) = a^n \sin(\omega_0 n) u(n)$ is

$$X(z) = \frac{a z^{-1} \sin\omega_0}{1 - 2az^{-1}\cos\omega_0 + a^2 z^{-2}}, \qquad |z| > |a|.$$