```
z_8 = conj(z_3)          % -0.5000 - 0.8660i
```

```
z = [z_1; z_2; z_3; z_4; z_5; z_6; z_7; z_8]; num = poly(z);
disp('FIR filter coefficients:'); disp(num);
```

The transfer function will be:

$$\begin{aligned} H(z) &= \prod_{k=1}^{8}\left(1 - z_k z^{-1}\right) = 1 + 1.1353z^{-1} + 0.5635z^{-2} + 5.6841z^{-3} \\ &\quad +4.9771z^{-4} + 5.6841z^{-5} + 0.5635z^{-6} + 1.1353z^{-7} + z^{-8}. \end{aligned}$$

7.4 Exercises

1. Design a 21 FIR LPF, with the cutoff frequency of 0.2.
2. Redesign the previous filter for a larger transition band, but the same order.
3. Graph the rectangular, triangular, Blackman, Hamming, Hanning, Kaiser (different β) windows, and also their magnitude frequency response characteristics. Note the principal lobes' values and the maximum amplitudes of the secondary lobes [dB]. Verify the results by those given in Tab. 7.1. For $N = 20$ study `demowindows.m`. What happens if you modify the window's length ($N = 50$)? What can you say about the windows effect in the FIR filters design?
4. Design an FIR band-reject filter, of length 21, using the windowing technique (use the windows generated in exercise 3), with the stop band limits $F_{s1} = 10$ kHz, $F_{s2} = 15$ kHz. The sampling frequency considered is $F_s = 90$ kHz. Plot the frequency response characteristics, the zeros distribution and the impulse response of the filter, for each window.
5. Design an FIR LPF, of order 36, using the sampling in the frequency-domain method, with the pass band limit $F_p = 15$ kHz and the sampling frequency $F_s = 50$ kHz. Sketch the frequency characteristics of the filter, the zeros distribution and the impulse response.
6. Design an FIR BPF, of minimum order, with the frequencies $F_{s1} = 10$ kHz, $F_{p1} = 12$ kHz, $F_{p2} = 60$ kHz, $F_{s2} = 62$ kHz and the sampling frequency $F_s = 130$ kHz, the minimum attenuation in the stop bands 40 dB and the maximum attenuation in the pass band 3 dB. Sketch the frequency characteristics of the filter.

Hint: In order to evaluate the pass and stop band deviations use:

$$A_{PB} = 20\lg\frac{1+\delta_p}{1-\delta_p} \Leftrightarrow \delta_p = \frac{10^{\frac{A_{PB}}{20}}-1}{10^{\frac{A_{PB}}{20}}+1}$$

$$A_{SB} = -20\lg\delta_s \Leftrightarrow \delta_s = 10^{-\frac{A_{SB}}{20}}.$$

7. Design a 33 order Hilbert transformer, with optimum response, such that the normalized frequency to be within 0.05 and 0.45.

8. Design a 55 linear-phase FIR LPF, with the transition frequencies 0.2 and 0.3.

9. Design an FIR filter that approximates the magnitude characteristic:

$$|H(\omega)| = \begin{cases} 0, & 0 < \omega < 0.2\pi, \\ 1, & 0.25\pi < \omega < 0.45\pi \\ 0, & 0.5\pi < \omega < \pi. \end{cases}$$

   Evaluate and sketch the impulse response and the frequency characteristics.

10. Design a linear-phase LPF, of order 51, to approximate the characteristic of an ideal LPF. The cutoff frequency is considered to be 0.2π. Graph the frequency characteristics for the designed filter. You should observe the presence of the Gibbs phenomenon.

11. Design a linear-phase BPF, of order 40, to approximate the characteristic of an ideal BPF (rectangular window) with the cutoff frequencies: 0.2π and 0.6π.

12. Repeat the previous exercise for a Hamming window. Why the Gibbs phenomenon doesn't appear anymore? What can you say about the transition band?

13. Design a 40 order filter to approximate the characteristic of a differentiator, $H(\omega) = \omega/\pi$, considering a Blackman window.

14. Consider the moving average filter described by the finite-coefficients difference equation:

$$y(n) = \frac{1}{3}[x(n) + x(n-1) + x(n-2)].$$

   - Evaluate and plot the magnitude and the log-magnitude frequency response.
   - At the input of this filter a signal mixed up with noise is applied. What do you obtain at the output? Comment on the result.

- Repeat the previous parts for moving average filter described by:

$$y(n) = \frac{1}{5}\left[x(n) + x(n-1) + x(n-2) + x(n-3) + x(n-4)\right].$$

15. Design an equiripple 55 order FIR filter, to approximate the frequency response:

$$H(\omega) = \begin{cases} 0, & 0 < \omega < 0.2\pi, \\ 1, & 0.22\pi < \omega < 0.43\pi \\ 0, & 0.5\pi < \omega < \pi. \end{cases}$$

16. Using a rectangular window, design an FIR BPF, of order 55, with the normalized cutoff frequencies 0.18 and 0.33. Plot the impulse response and the frequency response characteristics.