may be set to any arbitrary value (e.g. $T = 1$ s). The next example illustrates this point.

6. **Design of a digital LPF from an analog one by means of bilinear transform**. We need to design a single-pole IIR LPF, with the cutoff frequency 0.3π, by means of the bilinear transform, starting from an analog LPF with the system function:
$$H_a(s) = \frac{\Omega_p}{s+\Omega_p},$$
where Ω_p is the 3-dB bandwidth of the analog filter.
In the analog domain, the frequency $\omega_p = 0.3\pi$ corresponds to:
$$\Omega_p = \frac{2}{T}\tan\frac{\omega_p}{2} = \frac{2}{T}\tan\frac{0.3\pi}{2} = \frac{1.02}{T}.$$
Thus the analog filter has the system function:
$$H_a(s) = \frac{\frac{1.02}{T}}{s+\frac{1.02}{T}}.$$
By applying the bilinear transformation, we obtain:
$$H(z) = \frac{\frac{1.02}{T}}{\frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}}+\frac{1.02}{T}} = \frac{1.02(1+z^{-1})}{3.02-0.98z^{-1}} = \frac{0.3377(1+z^{-1})}{1-0.3245z^{-1}}.$$
The frequency response of the digital filter is:
$$H(\omega) = \frac{0.3377(1+e^{-j\omega})}{1-0.3245e^{-j\omega}}.$$
At $\omega = 0$, $H(0) = 1$, and at $\omega = 0.3\pi$, we have $|H(0.3\pi)| = 0.707$, which is the desired response.

9.4 Exercises

1. Using the impulse invariance method design a digital Butterworth BPF, for which:
   - The attenuation is lower than 1 dB at 4 kHz and 6 kHz;
   - The attenuation is greater than 40 dB at 3 kHz and 8 kHz;
   - The sampling frequency is 20 kHz.

   (a) Evaluate and sketch the frequency response characteristics for the analog BPF and for the corresponding digital one;

(b) Evaluate and sketch the impulse response of the digital BPF and the pole-zero diagram in the z-plane;

(c) Is the obtained digital filter stable?

Redesign the filter using the bilinear transformation method.

2. Repeat exercise 1 for a Cebyshev type I filter. Comment on the differences.

3. Repeat exercise 1 for a Cebyshev type II filter. Comment on the differences.

4. Repeat exercise 1 for an elliptic filter. Comment on the differences.

5. Consider the analog filter described by the system function:
$$H(s) = \frac{2}{s+2} \frac{s^2+2}{2s^2+3s+2}.$$
Using the bilinear transformation method, obtain the corresponding digital filter; the sampling period is equal by $T = 0.8$. What kind of filter is the obtained one? Is this filter stable?

6. Design a 5 order Butterworth LPF, which satisfies the condition: $0.9 < H(\omega) < 1$, for $0 < f < 0.2$.

7. Design a minimum order Butterworth LPF, which satisfies the conditions: $0.99 < |H(f)| < 1$ for $0 < f < 0.22$, and $0 < |H(f)| < 0.01$ for $0.25 < f < 0.5$.
   - Plot the frequency response characteristics (the magnitude, the phase and the group delay);
   - Find the poles and zeros of the system function and write the system function in a compact manner.

8. Repeat the previous exercise for a Cebyshev filter.

9. Design a Cebyshev BRF which must reject the frequency $f = 0.22$. The design must satisfy the next requirements:
   - The order of the filter is ten;
   - The stopband width is 0.04;
   - The transition bands are 0.03;
   - The stopband attenuation must be at least 20 dB;
   - The passband ripple is 1 dB.

   Evaluate the output of this filter to the excitation $x(n) = \sin(2\pi 0.22n)$, for $n = \overline{0, 299}$. Comment on the result.