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Timing Synchronization and Node Localization in Wireless Sensor Networks

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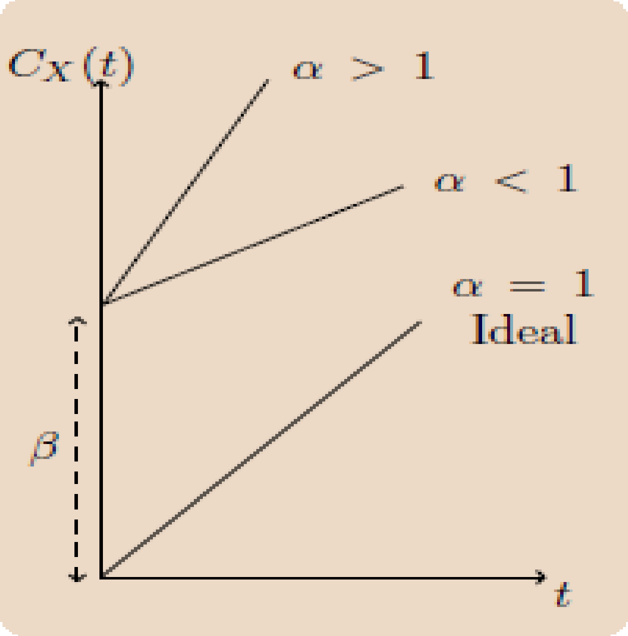
Wireless Sensor Networks

- Comprise large number of geographically distributed sensor nodes
- Nodes have sensing, computation and communication capabilities
- A Myriad of Applications
  - Battlefield surveillance
  - Environment and habitat monitoring
  - Industrial process control
  - Target localization and tracking
- Design Challenges
  - Limited hardware e.g., power, computational resources
  - Limited bandwidth
  - Low complexity, energy efficient inference algorithms
  - Scalability

Clock Synchronization

- Objective:
  To establish a common notion of time across the network.
- Why?
  - Efficient duty cycling
  - Optimal data fusion
  - Node localization and target tracking
  - Channel access schemes e.g., TDMA

Clock Model and Impairments



- Offset-only clock model

$$C_X(t) = t + \beta$$

- Offset and skew clock model

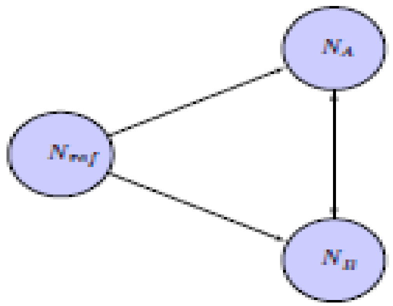
$$C_X(t) = \alpha t + \beta$$

t: actual time, α: Skew, β: Clock offset

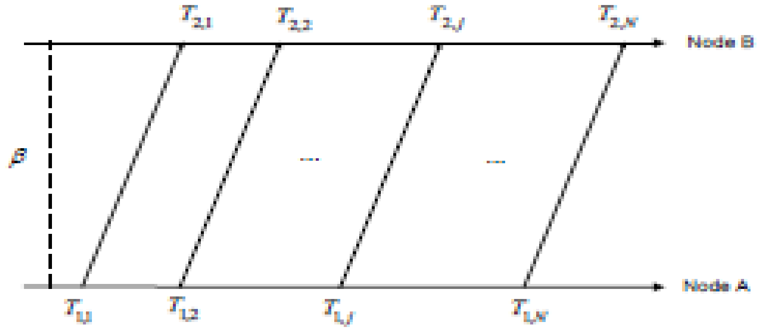
- Messages exchanged between nodes are contaminated by
  - A fixed propagation delay d
  - Variable network delays
  - Accurate modeling of the network delays is a topic of interest
  - Some of the candidate distributions include Gaussian, exponential, log-normal and Weibull (Bovy, 02).

Fundamental Approaches to Clock Synchronization

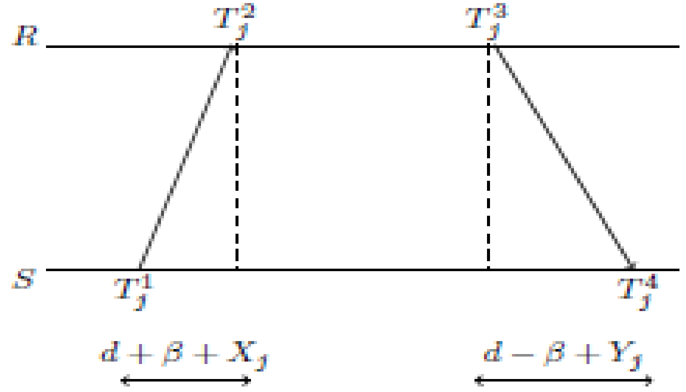
- Receiver-receiver synchronization
  - Nodes receiving timing information from a reference node synchronize by exchanging their time stamps.



- One way message exchange
  - Reference node broadcasts its current time to other nodes in the network.

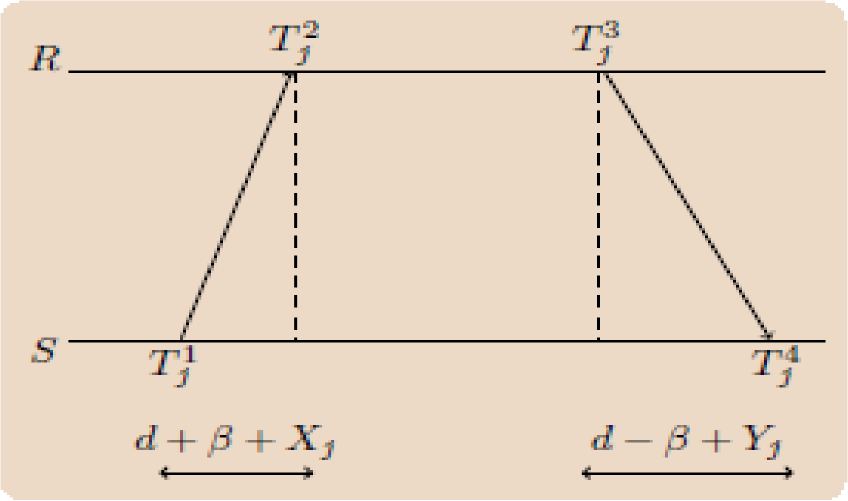


- Two way message exchange
  - Two nodes exchange their timing information with each other for synchronization



Pairwise Synchronization

- Node S sends its current time through time stamp T_j^1.
- Node R records the reception time T_j^2 according to its own time scale.
- Node R replies with time stamps T_j^2 and T_j^3 which is received at time T_j^4 by node S with respect to its own clock.



$$\begin{aligned} U_j &\triangleq T_j^2 - T_j^1 = d + \beta + X_j \\ V_j &\triangleq T_j^4 - T_j^3 = d - \beta + Y_j \end{aligned} \tag{1}$$

where d represents propagation delay, β is the clock offset and X_j and Y_j are the random delays.

Our primary goal is to estimate β using time stamps $\{U_j, V_j\}_{j=1}^N$.

Related Work

- Analysis of the sender receiver model. Several estimators of clock offset were proposed (Ghaffar, 2002)
- ML estimation of fixed delay d and clock offset β in a two way message exchange by graphical maximization (Jeske, 2005)

However.....

- Prior contributions obtain ML estimate of the clock parameters graphically.
  Need a simpler analytical framework.
- Most studies assume a fixed clock offset.
  Imperfect oscillators render a time-varying nature to the clock offset.
- Low-cost sensor nodes cannot afford complex iterative updates.
  Low complexity algorithms with exact inference.

Main Contributions

- A simpler alternative proof of the ML estimator proposed in (Jeske, 2005).
  Tool: Convex Optimization
- A unified ML estimation approach for Gaussian, log-normal or exponentially distributed likelihood functions
  Tool: Convex Optimization
- Recasting clock offset estimation in Bayesian regime to cater for time variations
  Tool: Factor Graphs
- An exact solution for time-varying clock offset estimation problem for Gaussian, log-normal or exponentially distributed likelihood functions
  Tool: Max-product message passing

Recasting in Convex Optimization Framework

- The problem of ML estimation can be recast as an instance of convex optimization.

$$(\hat{d}, \hat{\beta}) = \min_{d,\beta} \sum_{j=1}^{N} (U_j + V_j - 2d)$$
$$s.t\ U_{(1)} - d - \beta \geq 0, \quad V_{(1)} - d + \beta \geq 0.$$

Theorem

Using KKT conditions, the ML estimates of d and β are given by

$$\hat{\beta}_{ML} = \frac{U_{(1)} - V_{(1)}}{2}, \quad \hat{d}_{ML} = \frac{U_{(1)} + V_{(1)}}{2}$$

Simpler alternative bypassing the graphical analysis

A. Ahmad, A. Noor, E. Serpedin, H. Nounou and M. Nounou, "On Clock Offset Estimation in Wireless Sensor Networks with Weibull Distributed Network Delays", International Conference on Pattern Recognition *(ICPR)*, Istanbul, Turkey, August 2010.

A Parameterized Approach

- Aim is to provide an analytical parameterized solution for different distributions
- A general approach is used by considering the exponential family notation
- Define

$$\xi = d + \beta, \quad \psi = d - \beta \tag{2}$$

- Two types of likelihood functions: unconstrained and constrained

Unconstrained Likelihood:

$$f(\mathbf{U}_k|\xi) \propto \exp\left(\xi\eta_\xi(U_k) - \phi_\xi(\xi)\right)$$
$$f(\mathbf{V}_k|\psi) \propto \exp\left(\psi\eta_\psi(V_k) - \phi_\psi(\psi)\right)$$

Constrained Likelihood:

$$f(\mathbf{U}_k|\xi) \propto \exp\left(\xi\eta_\xi(U_k) - \phi_\xi(\xi)\right)\mathbb{I}\left(U_k - \xi\right)$$
$$f(\mathbf{V}_k|\psi) \propto \exp\left(\psi\eta_\psi(V_k) - \phi_\psi(\psi)\right)\mathbb{I}\left(V_k - \psi\right)$$

A Unified ML Estimation Approach

- Unconstrained Likelihood:

Lemma

The ML estimates of ξ and ψ can be expressed as

$$\hat{\xi}_{ML} = \frac{\sum_{j=1}^{N} \eta_{\xi}(U_j)}{N\sigma^2_{\eta_\xi}}, \quad \hat{\psi}_{ML} = \frac{\sum_{j=1}^{N} \eta_{\xi}(V_j)}{N\sigma^2_{\eta_\psi}}$$

- Constrained Likelihood:

Lemma

The ML estimates of ξ and ψ can be expressed as

$$\hat{\xi}_{ML} = \min\left(\frac{\sum_{j=1}^{N} \eta_{\xi}(U_j)}{N\sigma^2_{\eta_\xi}}, U_{(1)}\right), \quad \hat{\psi}_{ML} = \min\left(\frac{\sum_{j=1}^{N} \eta_{\psi}(V_j)}{N\sigma^2_{\eta_\psi}}, V_{(1)}\right)$$

A unified analytical ML approach estimation for Gaussian, log-normal and exponentially distributed likelihood functions

A Bayesian Viewpoint

- The imperfections introduced by environmental conditions in the quartz oscillator results in a time-varying clock offset between nodes.
- The parameters ξ and ψ are assumed to evolve through a Gauss-Markov process given by

$$\xi_k = \xi_{k-1} + w_k, \quad \psi_k = \psi_{k-1} + v_k, \quad \text{for } k = 1, \ldots, N$$

The posterior pdf can be expressed as

$$f(\boldsymbol{\xi}, \boldsymbol{\psi}|\boldsymbol{U}, \boldsymbol{V}) \propto f(\boldsymbol{\xi}, \boldsymbol{\psi}) f(\boldsymbol{U}, \boldsymbol{V}|\boldsymbol{\xi}, \boldsymbol{\psi}) = f(\xi_0) \prod_{k=1}^{N} f(\xi_k|\xi_{k-1}) f(\psi_0) \prod_{k=1}^{N} f(\psi_k|\psi_{k-1}) \cdot \prod_{k=1}^{N} f(U_k|\xi_k) f(V_k|\psi_k) \quad (3)$$

where uniform priors $f(\xi_0)$ and $f(\psi_0)$ are assumed. Define $\delta_{k-1}^{k} \triangleq f(\xi_k|\xi_{k-1}) \sim \mathcal{N}(\xi_{k-1}, \sigma^2)$, $\nu_{k-1}^{k} \triangleq f(\psi_k|\psi_{k-1}) \sim \mathcal{N}(\psi_{k-1}, \sigma^2)$, $f_k \triangleq f(U_k|\xi_k)$, $h_k \triangleq f(V_k|\psi_k)$

Factor Graphs and Max-Product Message Passing

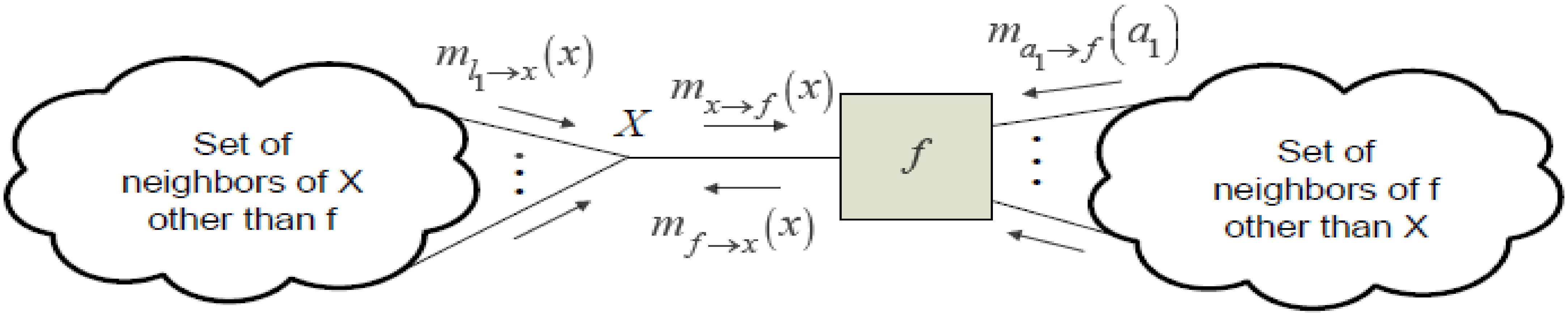
- A factor graph represents a factorization of a global function as a product of local functions called *factors*.
- An edge connects a variable to a factor node only if it is an argument of the local function factor expressed by the factor node.

variable to factor node:

$$m_{x \to f}(x) = \prod_{l_i \in n(x) \setminus f} m_{l_i \to x}(x)$$

factor node to variable:

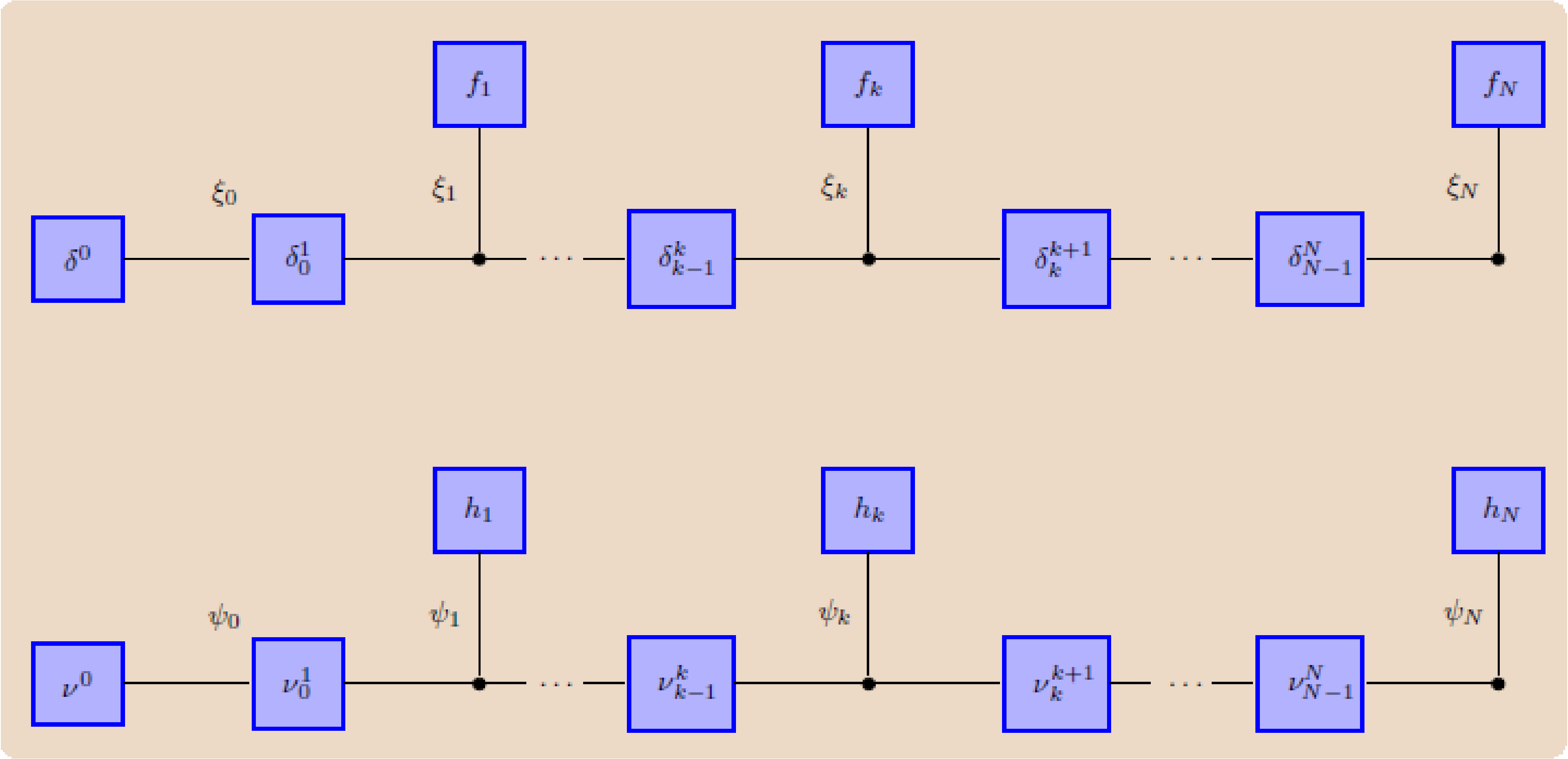
$$m_{f \to x}(x) = \max_{\setminus \{x\}} \left(f(.) \prod_{a_i \in n(f) \setminus \{x\}} m_{a_i \to f}(a_i) \right)$$



A. Ahmad, E. Serpedin, and K. Qaraqe, "Factor Graphs and Message Passing Algorithms", Mathematical Foundations for Signal Processing, Communications and Networking, CRC Press, Feb. 2012.

Factor Graph Representation

The factor graph representation of (3) is



- Cycle-free nature guarantees convergence of message passing.
- Analyze the graph of ξ only, calculations for ψ being analogous.

Exact Solution

Theorem

The estimate $\hat{\xi}_N$ can be expressed as

$$\hat{\xi}_N = \min\left(U_N, G_N^N\left(U_{N-1}\right), \ldots, G_2^N\left(U_1\right), G_1^N\left(\hat{\xi}_0\right)\right) \tag{4}$$

The factor graph-based estimate (FGE) of the clock offset $\hat{\beta}_N$ is then given by

$$\hat{\beta}_N = \frac{\hat{\xi}_N - \hat{\psi}_N}{2} \, .$$

- Solution is applicable to cases when the likelihood $f(U_k|\xi_k)$ is Gaussian, log-normal or exponentially distributed.

A. Ahmad, D. Zennaro, E. Serpedin, and L. Vangelista, "A Factor Graph Approach to Clock Offset Estimation in Wireless Sensor Networks", *IEEE Transactions on Information Theory*, vol. 58, no. 7, pp. 4244-4260, July 2012.

A. Ahmad, D. Zennaro, E. Serpedin, and L. Vangelista, "Time Varying Clock Offset Estimation in Two-way Message Exchange in Wireless Sensor Networks using Factor Graphs", *ICASSP*, Kyoto, Japan, March 2012.

Performance Bounds - Contributions

- Classical as well as Bayesian lower bounds on the variance of the aforementioned estimators are derived.
- Valid for arbitrary distributions from the exponential family.
- Cramer-Rao bound (CRB) and Chapman-Robbins (CHRB) are derived for unconstrained and constrained likelihood functions, respectively.

Unconstrained Likelihood:

$$f(\mathbf{Z};\rho) \propto \exp\left(\rho \sum_{j=1}^{N} \eta(Z_j) - N\phi(\rho)\right)$$

Constrained Likelihood:

$$f(\mathbf{Z};\rho) \propto \exp\left(\rho \sum_{j=1}^{N} \eta(Z_j) - N\phi(\rho)\right) \prod_{j=1}^{N} \mathbb{I}(Z_j - \rho)$$

A. Ahmad, D. Zennaro, E. Serpedin, and L. Vangelista, "A Factor Graph Approach to Clock Offset Estimation in Wireless Sensor Networks", *IEEE Transactions on Information Theory*, vol. 58, no. 7, pp. 4244-4260, July 2012.

Classical Bounds

Lemma

The CRB for ρ in the unconstrained likelihood function is given by

$$\mathrm{Var}(\hat{\rho}) \geq \frac{1}{N\sigma_\eta^2}, \qquad \sigma_\eta^2 = \frac{\partial^2 \phi(\rho)}{\partial \rho^2}.$$

Lemma

The CHRB for the parameter ρ given the constrained likelihood function can be expressed as

$$\mathrm{Var}(\hat{\rho}) \geq \left[\inf_h \frac{\left\{ (M_\eta(h))^{-2N} \cdot \zeta^N(h) - 1 \right\}}{h^2} \right]^{-1}$$

where $M_\eta(h)$ is the MGF of the statistic $\eta(Z_j)$ and

$$\zeta(h) \triangleq \mathbb{E}\left[\exp\left(2h\eta(Z_j)\right) \mathbb{I}\left(Z_j - \rho - h\right)\right]$$

with the expectation taken with respect to any Z_j.

Bayesian Bounds

Lemma

The Bayesian CRB states that

$$\mathrm{Var}\left(\hat{\rho}_k\right) \geq J_{\mathrm{CR}}^{-1}(k) \triangleq [\boldsymbol{J}_{\mathrm{CR}}^{-1}(k)]_{kk} \,.$$

where $J_{\mathrm{CR}}^{-1}(k)$ is the Bayesian information matrix. For our Gauss-Markov evolution model, it follows that

$$J_{\mathrm{CR}}(k+1) = \left(\sigma^2 + J_{\mathrm{CR}}^{-1}(k)\right)^{-1} + \sigma_{\eta_k}^2$$

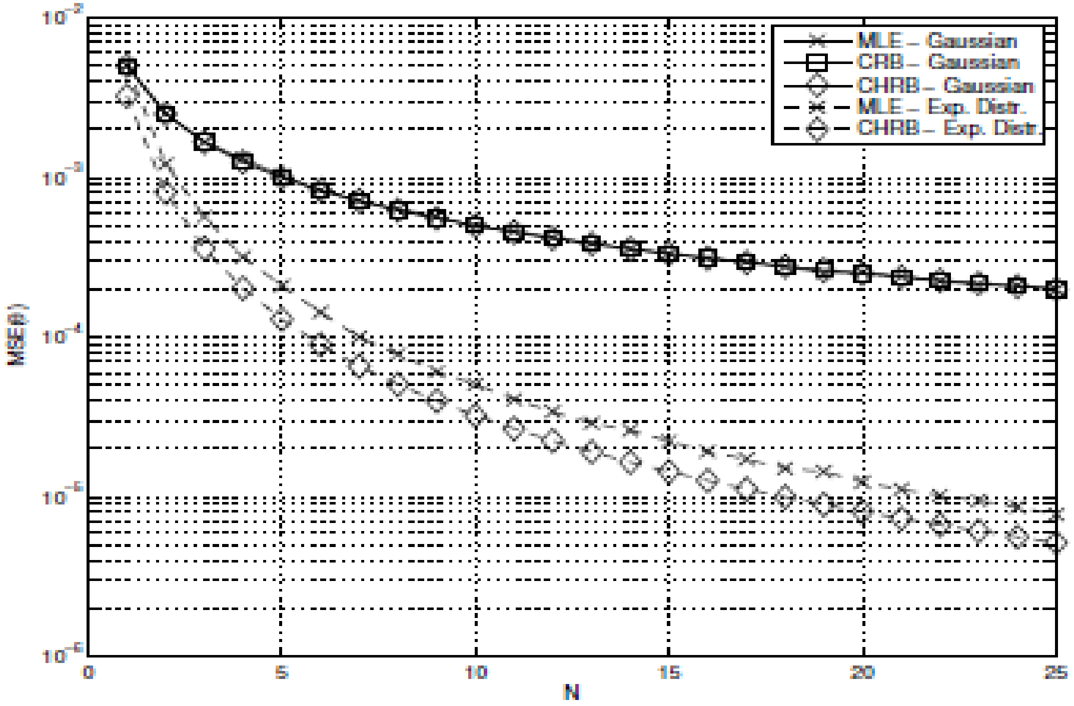
Lemma

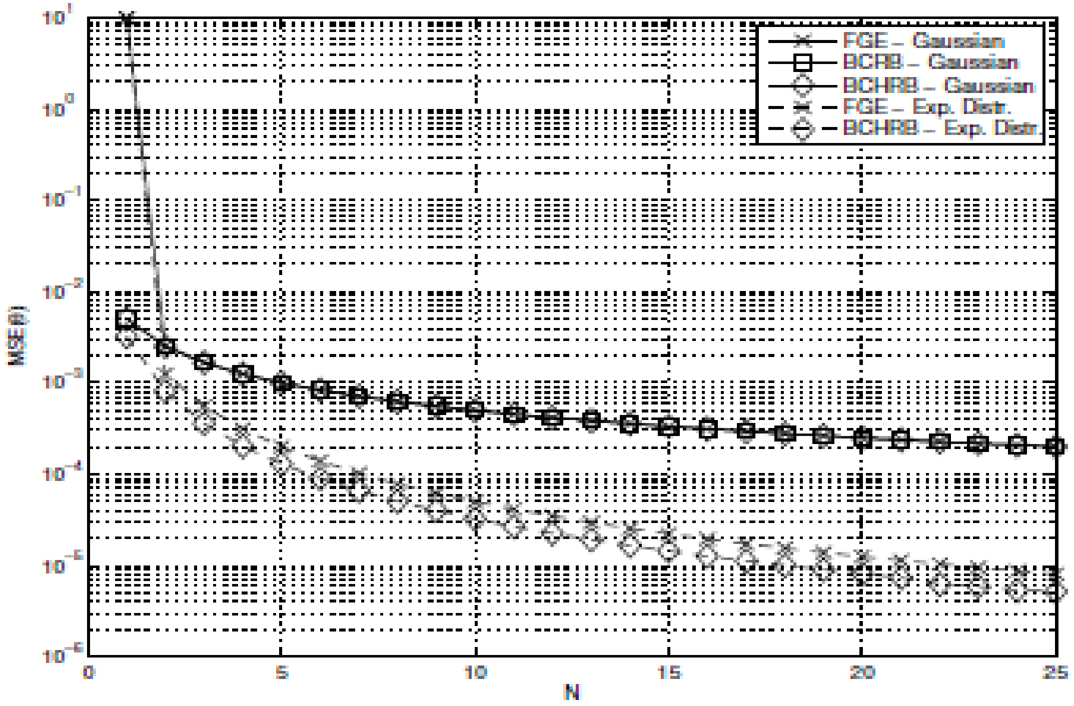
The BCHRB for the parameter ρ_k can be expressed as

$$\mathrm{Var}(\hat{\rho}_k) \geq \frac{1}{J_{CH,k}}$$

$$J_{CH,k} = \inf_{\mathbf{h}_k} \frac{T_k(\mathbf{h}_k) - 1}{h_k^2}, \quad T_k(\mathbf{h}_k) = \left(\prod_{j=1}^{k} M_\eta^{-2}(h_j) M_\eta(2h_j)\right) \exp\left[\sum_{j=1}^{k} \frac{(h_j - h_{j-1})^2}{\sigma^2}\right] .$$

Simulation Results



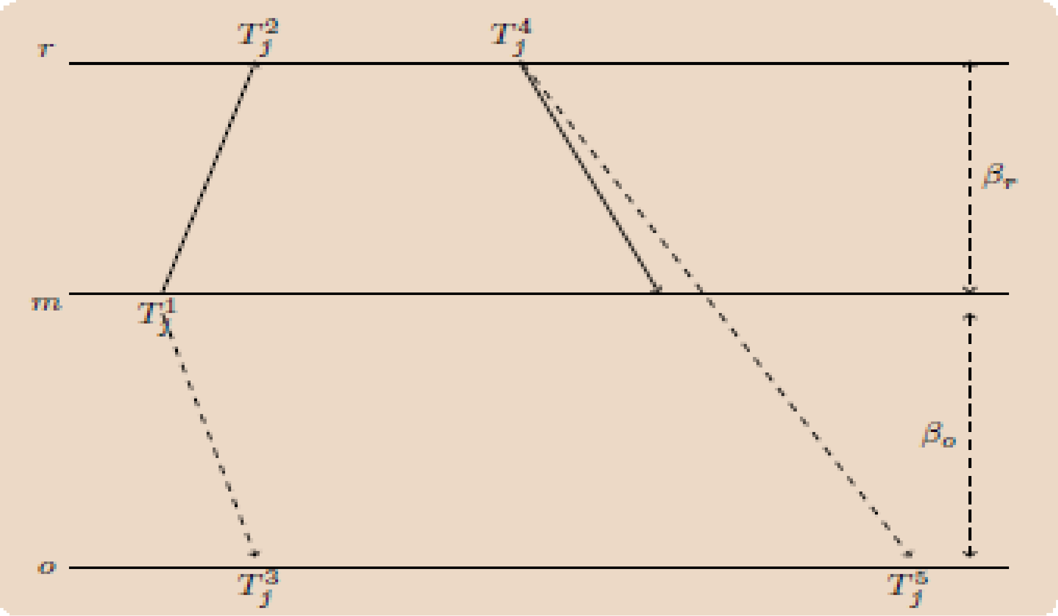


- MSE decreases with increasing number of message exchanges.
- In case of Gaussian likelihood, the lower bound is achieved, while for exponentially distributed likelihood function, lower bound is not achieved.
- MSE for exponentially distributed likelihood function decays faster (at a rate of $1/N^2$ as compared to $1/N$ in Gaussian case).

Inactive Node Synchronization

Inactive nodes can synchronize themselves by overhearing the message exchanges between the reference node and another receiver node.

- Node m transmits its timing information to node r
- Node r replies with its own timing packet to m
- A node o overhears the timing messages exchanged between nodes m and r



$$
\begin{aligned}
U_j &\triangleq T_j^2 - T_j^1 = d + \beta_r + X_j \\
V_j &\triangleq T_j^3 - T_j^1 = d + \beta_o + Y_j \\
W_j &\triangleq T_j^5 - T_j^4 = d + \beta_o - \beta_r + Z_j
\end{aligned} \tag{5}
$$

- For X_j, Y_j and Z_j i.i.d exponentially distributed, the ML estimates d, β_r and β_o were proposed in (Chaudhari, 2010) using a complex graphical search.

Contribution - An Analytical Proof

- The problem of ML estimation can be recast as an instance of convex optimization.

$$(\hat{d}, \hat{\beta}_r, \hat{\beta}_o) = \min_{d,\beta_r,\beta_o} \sum_{j=1}^{N} (U_j + V_j + W_j - 2\beta_o - 3d)$$

$$s.t\, U_{(1)} - d - \beta_r \geq 0, \quad V_{(1)} - d + \beta_o \geq 0, \quad W_{(1)} - d - \beta_o + \beta_r \geq 0$$

Theorem

Using KKT conditions, the offset and propagation delay estimates are given by

$$\hat{d} = U_{(1)} + W_{(1)} - V_{(1)}, \quad \hat{\beta}_r = V_{(1)} - W_{(1)}, \quad \hat{\beta}_o = 2V_{(1)} - U_{(1)} - W_{(1)}$$

Simpler alternative bypassing the graphical analysis.

A. Ahmad, A. Noor and E. Serpedin, "Joint Clock Offset and Skew Estimation for Inactive Nodes in Wireless Sensor Networks", Conference on Information Sciences and Systems *(CISS)*, Maryland, USA, March 2011.

Contribution - A Bayesian Viewpoint

- A Bayesian approach, similar to the one for pairwise synchronization, can be used for clock estimation for inactive nodes.
- By defining $\xi = d + \beta_r$, $\psi = d + \beta_o$ and $\zeta = d + \beta_o - \beta_r$, (5) becomes

Inactive Node Synchronization

$$U_k = \xi_k + X_k$$
$$V_k = \psi_k + Y_k$$
$$W_k = \zeta_k + Z_k$$

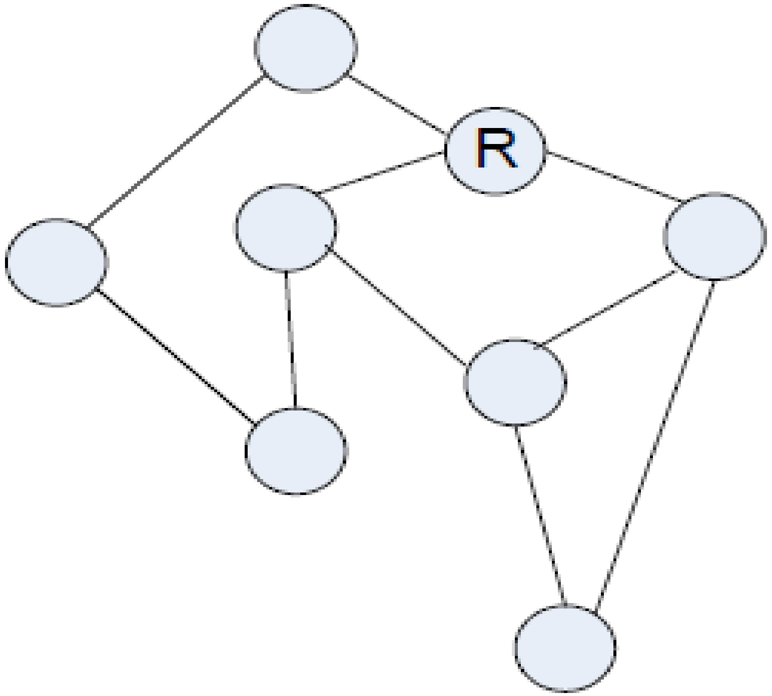
Pairwise Synchronization

$$U_k = \xi_k + X_k$$
$$V_k = \psi_k + Y_k$$

- Notice that the factor graph in this case will have precisely the same structure, just a redefinition of ξ_k, ψ_k and ζ_k.
- Our message updates based on max-product will result in an exact solution for the case of inactive node synchronization as well.

Distributed Network-wide Synchronization

- A natural extension of the aforementioned discussion is network-wide synchronization.
- Each node attempts to synchronize itself with a reference node by exchanging messages with neighbors.
- A distributed algorithm will enable a node to estimate its own clock offset as opposed to centralized processing.



Related Work

- Distributed network-wide synchronization algorithm by exploiting the natural network constraint that the relative clock offsets in network loops sum to zero (Borkar, 2006).
- A synchronization algorithm by assuming no initial clock offsets but time-varying skews among the oscillators (Freris, 2009).
- A Laplacian-based algorithm for establishing agreement on oscillation frequencies based on standard consensus (Simeone, 2007).
- Recently, distributed network-wide synchronization proposed using belief propagation for Gaussian distributed network delays (Leng, 2011).

Contributions

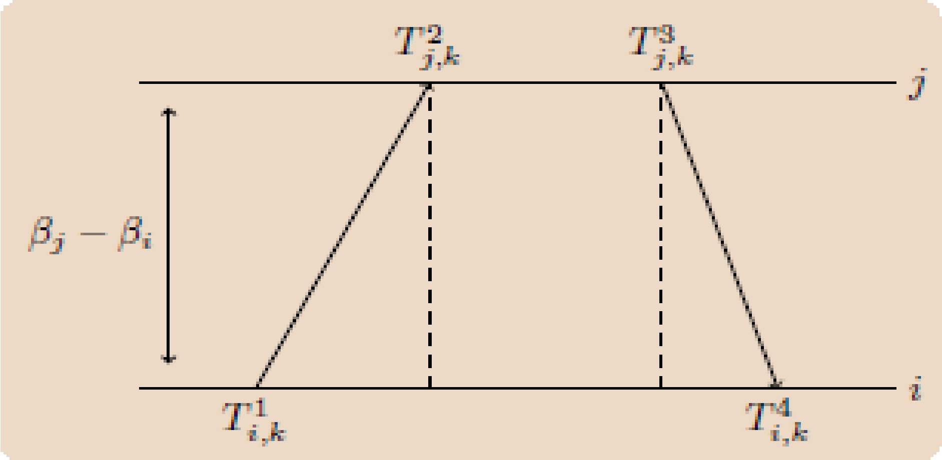
- A network-wide clock synchronization algorithm is proposed in case of exponentially distributed network delays by representing the sensor network as a factor graph.
- Inference is performed using max-product message passing algorithm.
- A closed form solution is obtained for the belief of each node about its clock offset.
- The proposed algorithm is fully distributed since the clock offset of each node is determined at the node itself, instead of centralized processing.

D. Zennaro, A. Ahmad, E. Serpedin, L. Vangelista, “Network-Wide Clock Synchronization via Message Passing with Exponentially Distributed Link Delays", IEEE Transactions on Communications, vol. 61, no. 5, May 2013.

System Model

- Building block is the pairwise message exchange discussed earlier.

$$U_{ij,k} \triangleq T^2_{j,k} - T^1_{i,k} = d_{ij} + (\beta_j - \beta_i) + X_{ij,k}$$
$$V_{ij,k} \triangleq T^4_{i,k} - T^3_{j,k} = d_{ij} - (\beta_j - \beta_i) + Y_{ij,k}$$
$$X_{ij,k} \sim \mathcal{E}(1/\lambda), \quad Y_{ij,k} \sim \mathcal{E}(1/\lambda)$$



- $[U_{ij,(1)}, V_{ij,(1)}]^T$ constitutes a sufficient statistics for estimating $(\beta_j - \beta_i)$

$$S_{ij} = (\beta_j - \beta_i) + Z_{ij} \tag{6}$$

- Consequently, $S_{ij} \sim \mathcal{L}\left(\beta_j - \beta_i, \frac{1}{2K\lambda}\right)$, so that

$$p\left(S_{ij}|\beta_i, \beta_j\right) = K\lambda \exp\left(-2K\lambda|S_{ij} - \beta_j + \beta_i|\right) .$$

Clock Offset Inference

- Our goal is to infer β_i for all i, using data S_{ij} gathered from the message exchanges between pairs of nodes $(i,j) \in L$.
- Inference about β_i can be obtained by

$$\hat{\beta}_i \triangleq \arg\max_{\beta_i} p_i\left(\beta_i | \boldsymbol{S}\right)$$

where the *a-posteriori* (AP) pdf, $p_i\left(\beta_i | \boldsymbol{S}\right)$, is given by

$$p_i\left(\beta_i | \boldsymbol{S}\right) = \int_{\tilde{\boldsymbol{\beta}}_i} p\left(\boldsymbol{\beta} | \boldsymbol{S}\right) \mathrm{d}\tilde{\boldsymbol{\beta}}_i \tag{7}$$

where $\tilde{\boldsymbol{\beta}}_i \triangleq [\beta_1, \ldots, \beta_{i-1}, \beta_{i+1}, \ldots, \beta_N]$.

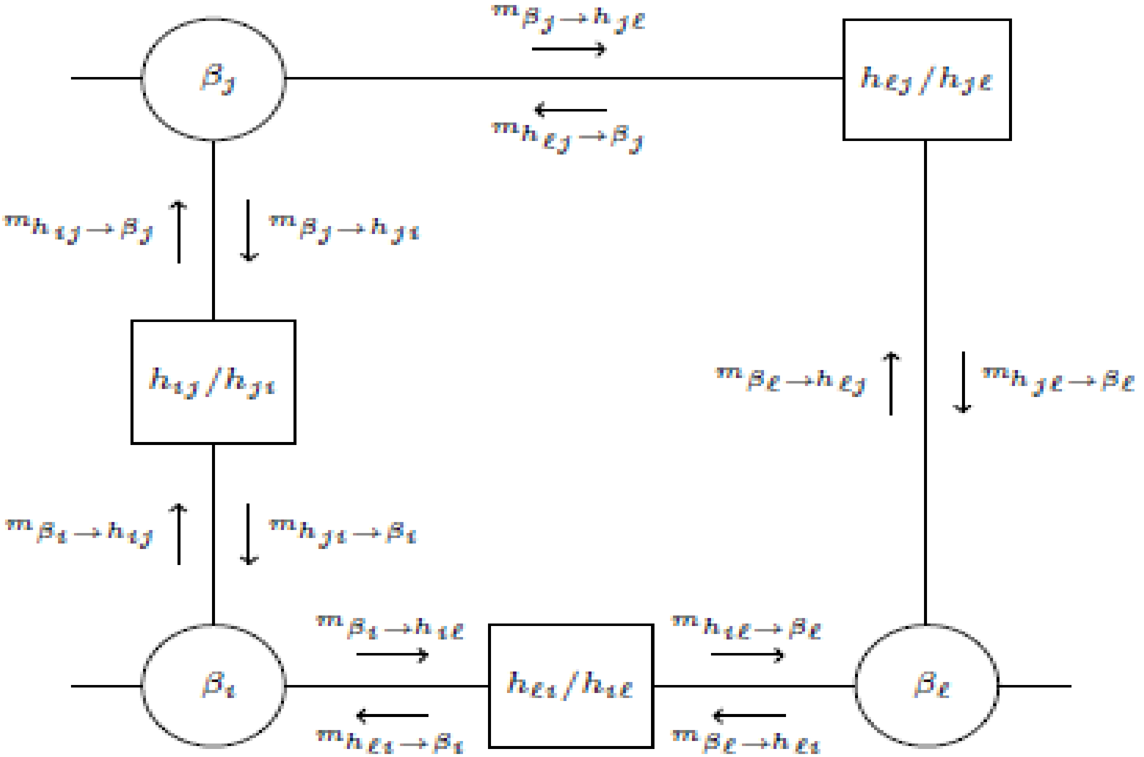
- Obtain the AP pdf using factor graphs and message passing.

Max-product Message Passing

$$m_{\beta_i \to h_{i\ell}}(\beta_i) = p_i(\beta_i) \cdot \prod_{j \in \mathcal{N}_i, j \neq \ell} m_{h_{ji} \to \beta_i}(\beta_i)$$

$$m_{h_{i\ell} \to \beta_\ell}(\beta_\ell) = \max_{\beta_i} \left[m_{\beta_i \to h_{i\ell}}(\beta_i)\, h_{i\ell}(\beta_i, \beta_\ell) \right]$$

$$b_i(\beta_i) = p_i(\beta_i) \prod_{j \in \mathcal{N}_i} m_{h_{ji} \to \beta_i}(\beta_i)$$



- The belief $b_i(\beta_i)$ represents the AP pdf.
- Maximization of the belief yields the clock offset at each node i.

$$\hat{\beta}_i = \arg\max_{\beta_i} b_i(\beta_i)$$

Message From Factor to Variable at Iteration t

Lemma

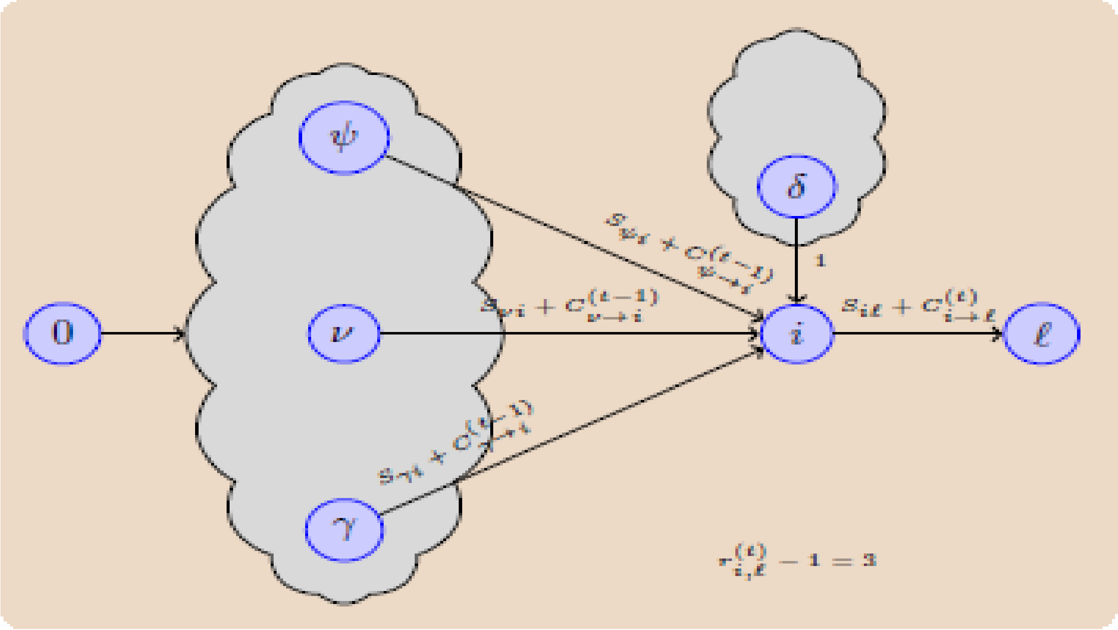
The message $m^{(t)}_{h_{i\ell}\to\beta_\ell}$ *can be approximated as*

$$m^{(t)}_{h_{i\ell}\to\beta_\ell}(\beta_\ell) \mathrel{\tilde{\propto}} \exp\left(-2K\lambda\left|\beta_\ell - \left(S_{i\ell} + C^{(t)}_{i\to\ell}\right)\right|\right), \tag{8}$$

$$C^{(t)}_{i\to\ell} = \begin{cases} W^{(t)}_{i\ell}\left(\frac{r^{(t)}_{i,\ell}}{2}\right), & \textit{even } r^{(t)}_{i,\ell}, \\ \frac{1}{2}\left[W^{(t)}_{i\ell}\left(\frac{r^{(t)}_{i,\ell}-1}{2}\right) + W^{(t)}_{i\ell}\left(\frac{r^{(t)}_{i,\ell}+1}{2}\right)\right], & \textit{odd } r^{(t)}_{i,\ell}. \end{cases} \tag{9}$$

where $\left(r^{(t)}_{i,\ell} - 1\right)$ *is the number of neighbors of node* i, *other than node* ℓ, *that have sent non-constant messages at iteration* $(t-1)$ *and the sequence* $\{W^{(t)}_{i\ell}(n)\}$ *denotes the non-constant messages sent by neighbors of node* i, *other than node* ℓ.

An Example



- The sequence $\{W^{(t)}_{i\ell}(n)\}$ is given by

$$\{W^{(t)}_{i\ell}(n)\} = \left\{S_{\gamma i} + C^{(t-1)}_{\gamma \to i}, S_{\nu i} + C^{(t-1)}_{\nu \to i}, S_{\psi i} + C^{(t-1)}_{\psi \to i}\right\} .$$

- Sort to obtain $W^{(t)}_{i\ell}(1)$, $W^{(t)}_{i\ell}(2)$ and $W^{(t)}_{i\ell}(3)$.
- The quantity $C^{(t)}_{i \to \ell}$ is then determined using (9).

Belief of Clock Offset at Node i

- At iteration t, belief $b_i^{(t)}$ is updated as

Theorem

$$b_i^{(t)}(\beta_i) = p_i(\beta_i) \cdot \prod_{j \in \mathcal{N}_i} m_{h_{ji} \to \beta_i}^{(t)}(\beta_i)$$
$$\propto \exp\left(-2K\lambda \sum_{n=1}^{r_i^{(t)}} \left|\beta_i - W_i^{(t)}(n)\right|\right), \quad (10)$$

where $W_i^{(t)}(n)$ is the sorted sequence of messages received from neighbors of node i.

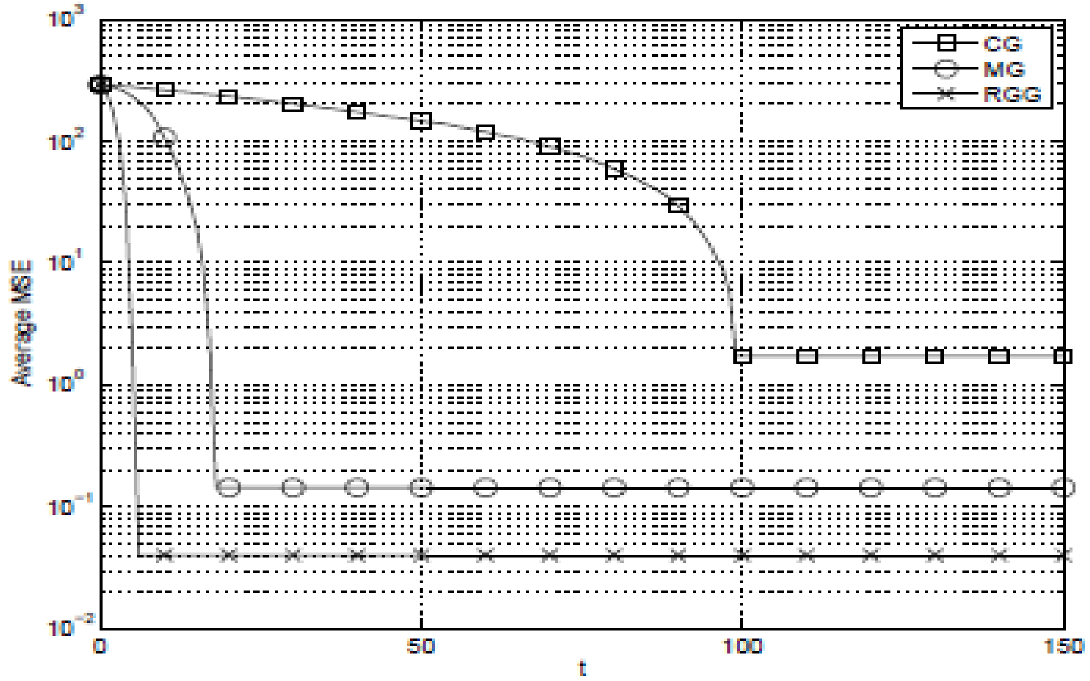
- Node i can compute the estimate $\hat{\beta}_i$ as follows

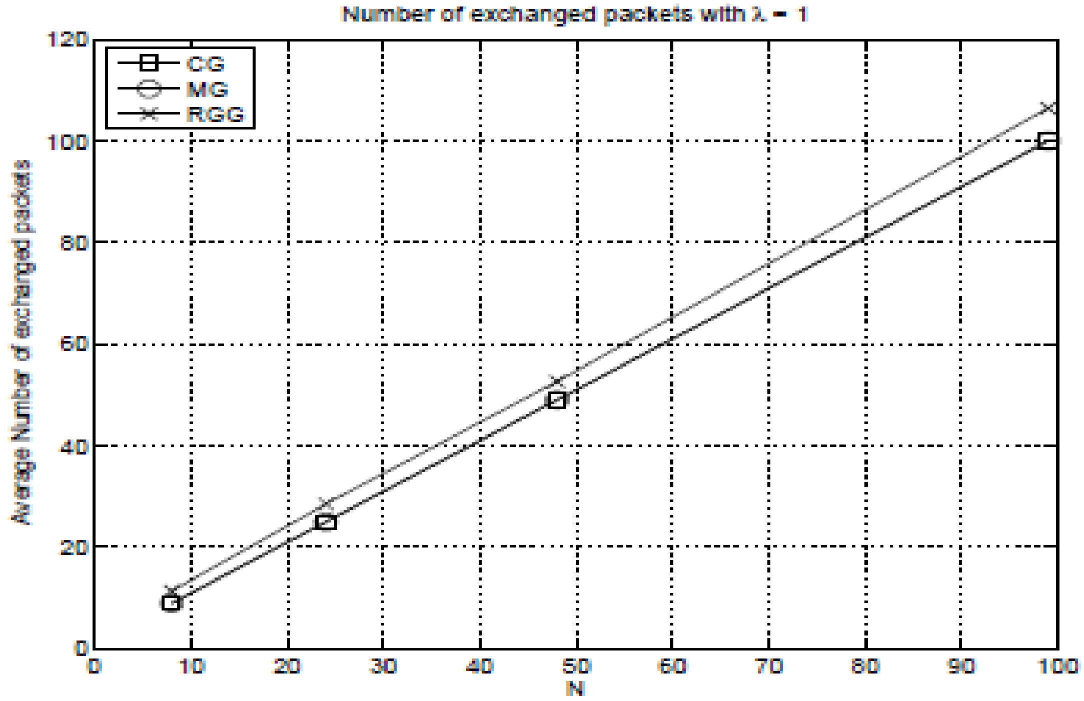
$$\hat{\beta}_i^{(t)} = \arg\max_{\beta_i} b_i^{(t)}(\beta_i) \quad (11)$$

A Synchronization Algorithm

$\mathcal{F} = \{0\}$.
For t **from** 0 **on**
Node $i = 0$ sends its timing information to its neighbors
For each node $i \notin \mathcal{F}$ **in parallel to node** 0
Node i computes the belief $b_i^{(t)}(\beta_i)$ according to (10) from data $W_i^{(t)}(n)$ received from its neighbors.
Node i computes the estimate $\hat{\beta}_i^*$.
If $|\hat{\beta}_i^* - \hat{\beta}_i^{(t-1)}|/\hat{\beta}_i^{(t-1)} > \varepsilon$
$\hat{\beta}_i^{(t)} = \hat{\beta}_i^*$.
Node i transmits all the messages $S_{i\ell} + C_{i\to\ell}^{(t)}$, computed according to (8) and (9), to its neighbors ℓ.
Else
$\hat{\beta}_i^{(t)} = \hat{\beta}_i^{(t-1)}$
$\mathcal{F} = \mathcal{F} \cup \{i\}$.
End If End For
For each node $i \in \mathcal{F}$ **in parallel to node** 0
$\hat{\beta}_i^{(t)} = \hat{\beta}_i^{(t-1)}$.
End For
End For

Simulation Results





- MSE decreases at the fastest rate when the underlying graph is random geometric (RGG) as compared to chain graph (CG) and mesh grid (MG).
- The complexity of the algorithm is almost linear in terms of number of packets exchanged for all three topologies considered.
- Suitable for implementation in a WSN.

Joint Localization and Synchronization (JLS)

- Several WSN applications such as geographical routing, disaster rescue, etc., require location awareness.
- Node localization in WSNs has been extensively studied.
- Time of arrival (TOA), time difference of arrival (TDOA) and received signal strength (RSS) are used for node localization.
- Since, TOA and TDOA are time-based techniques, synchronization is an important prerequisite in node localization as well.
- This close connection necessitates a joint estimation approach.

Related Work

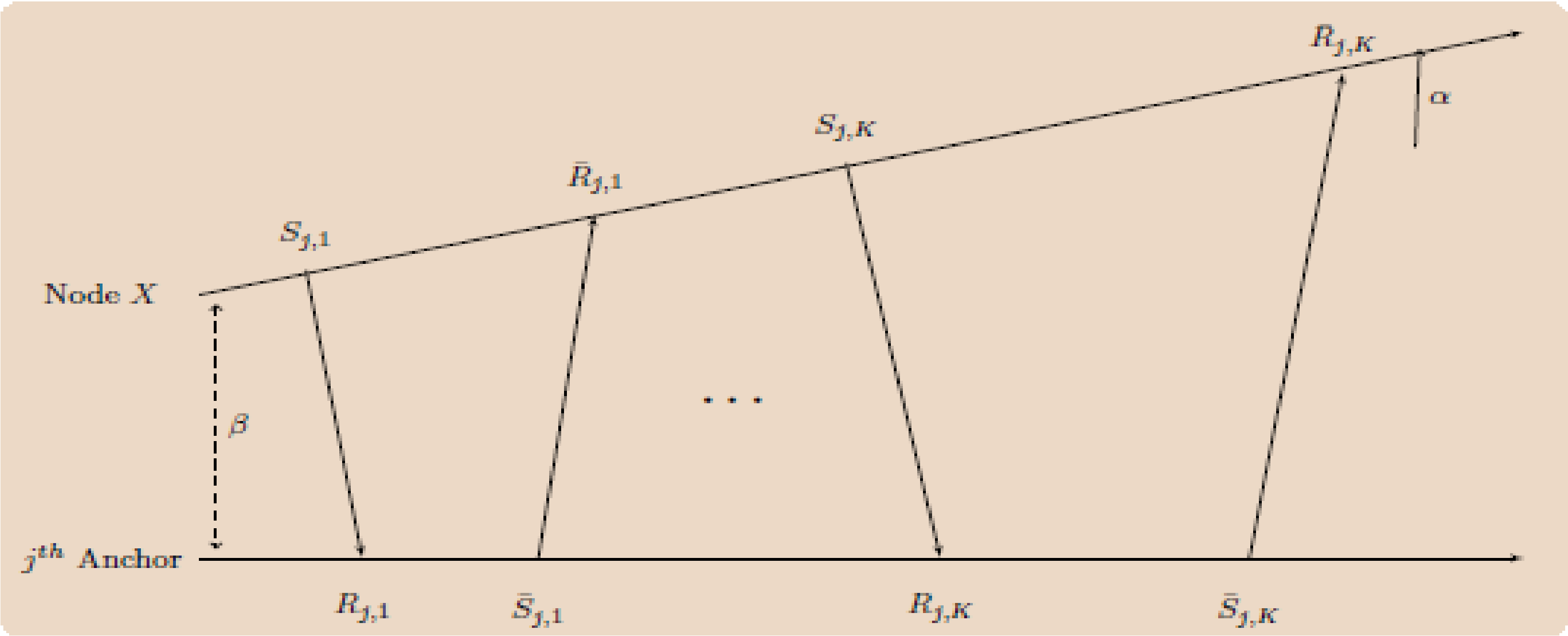
- Joint localization and synchronization in WSNs has received a lot of interest recently.
- Optimal and sub-optimal algorithms for estimating an unknown node's position and fixed clock parameters were derived in (Zheng, 2010).
- A weighted least squares approach for joint estimation is devised in (Zhu 2010) for fixed clock parameters.
- Several synchronization-only approaches have considered time-variations in clock parameters in WSNs (Chaudhari, 2010, Kim, 2012, Ahmad, 2012).
- Important to incorporate time variations to reduce re-synchronization requests.

Main Contributions

- We introduce the idea of temporal variations in clock parameters for joint localization and time-varying clock synchronization.
- A simple algorithm proposed to iteratively estimate the time-varying clock parameters and the unknown node's location.
  Tool: Expectation-Maximization (EM), Kalman Smoother.
- The M-step is further simplified by linearizing the data to estimate location.
  Tool: Least Squares (LS).
- Performance is benchmarked by deriving the Hybrid Cramér-Rao bound (HCRB).

A. Ahmad, E. Serpedin, H. Nounou and M. Nounou, ``Joint Node Localization and Time-Varying Clock Synchronization in Wireless Sensor Networks," IEEE Transactions on Wireless Communications, vol. 12, no. 10, Oct. 2013.

System Model



$$S_{j,k} = \alpha\left(R_{j,k} - d_j - w_{j,k}\right) + \beta$$
$$\bar{R}_{j,k} = \alpha\left(\bar{S}_{j,k} + d_j + \bar{w}_{j,k}\right) + \beta \ ,$$

where $d_j = \|x - s_j\|$, node location $x = [x_1 \ x_2]^T$, j^{th} anchor location s_j.

$$\text{Substitute} \qquad \theta_1 \triangleq \frac{1}{\alpha}, \qquad \theta_2 \triangleq \frac{\beta}{\alpha} \ , \tag{12}$$

Incorporating Temporal Variations

- We assume that θ evolves according to a Gauss-Markov process

$$\boldsymbol{\theta}_k = \boldsymbol{\theta}_{k-1} + \boldsymbol{n}_k \ ,$$

- Collecting messages exchanged with all anchors at the k^{th} instant, we have

$$\boldsymbol{y}_k - \boldsymbol{d}\left(\boldsymbol{x}\right) = \boldsymbol{H}_k \boldsymbol{\theta}_k + \boldsymbol{w}_k \ .$$

- The joint distribution of $\{\boldsymbol{y}, \boldsymbol{\Theta}\}$, parameterized by $\boldsymbol{x}$, can be expressed as

$$\begin{aligned} f(\boldsymbol{y}, \boldsymbol{\Theta}; \boldsymbol{x}) &= f\left(\boldsymbol{\Theta}\right) f\left(\boldsymbol{y}|\boldsymbol{\Theta}; \boldsymbol{x}\right) \\ &= f\left(\boldsymbol{\theta}_0\right) \prod_{k=1}^{K} f\left(\boldsymbol{\theta}_k|\boldsymbol{\theta}_{k-1}\right) \prod_{k=1}^{K} f\left(\boldsymbol{y}_k|\boldsymbol{\theta}_k; \boldsymbol{x}\right) \ . \end{aligned}$$

- Need simpler estimators for $(\boldsymbol{\Theta}, \boldsymbol{x})$ than the costly MAP estimator.

Expectation-Maximization (EM) Algorithm

- Iterative method used to determine the ML estimate of the parameters of a given distribution from incomplete data (Dempster, 1977).
- **E-Step**:
  Given $\hat{\boldsymbol{x}}^{(i)}$ and $\boldsymbol{y}$, determine the likelihood function

$$Q\left(\boldsymbol{x}, \hat{\boldsymbol{x}}^{(i)}\right) \triangleq \mathbb{E}_{\boldsymbol{\Theta}|\boldsymbol{y},\hat{\boldsymbol{x}}^{(i)}}\left[\ln f\left(\boldsymbol{z};\boldsymbol{x}\right)\right] \ . \tag{13}$$

- **M-Step**:
  Obtain an estimate of $\boldsymbol{x}$ at iteration index $i+1$ as follows

$$\hat{\boldsymbol{x}}^{(i+1)} = \arg\max_{\boldsymbol{x}} Q\left(\boldsymbol{x}, \hat{\boldsymbol{x}}^{(i)}\right) \ . \tag{14}$$

- Given $\hat{\boldsymbol{x}}^{(i)}$, the system is a linear Gaussian state space model in $\boldsymbol{\Theta}$.
- Minimum mean square error (MMSE) estimator $\hat{\boldsymbol{\theta}}^{(i)}_{k|K}$ can be obtained from a standard Kalman smoother.

E and M-Steps

- **E-Step** (Kalman Smoother):
  - Forward Recursion
  - Backward Recursion

- **M-Step**

  After some simplification, it can be shown that

$$\hat{\boldsymbol{x}}^{(i+1)} = \arg\min_{\boldsymbol{x}} \sum_{k=1}^{K} \left\| \tilde{\boldsymbol{y}}_k(\boldsymbol{x}) - \boldsymbol{H}_k \hat{\boldsymbol{\theta}}_{k|K}^{(i)} \right\|^2 .$$

Algorithm 1 `The EM Algorithm`

1: Input time-stamps $\{S_{j,k}, R_{j,k}, \bar{S}_{j,k}, \bar{R}_{j,k}\}_{k=1}^{K}$ and anchor locations
2: Initialize $\hat{x}^{(0)}$.
3: **for** $k = 1, \ldots, K$ **do**
4:     Determine $Q\left(x, x^{(i)}\right)$ using the MMSE estimator $\hat{\theta}_{k|K}^{(i)}$
5: **end for**
6: Obtain ML estimate $\hat{x}^{(i+1)}$ by solving the norm minimization problem.
7: **return**

A Least Squares Alternative

- The location estimator in EM requires a potentially costly 2-D norm minimization problem.
- Need to find a simpler alternative.
- We can use the estimates of $\hat{\boldsymbol{\theta}}_k$ from the Kalman smoother.
- Non-linearly process the data and ignore the second order noise terms.

The refined LS location estimator $\hat{\boldsymbol{x}}^{(i+1)}$ at iteration $i+1$ is given by

$$\hat{\boldsymbol{x}}_{\mathrm{LS}}^{(i+1)} = \left(\bar{\boldsymbol{M}}^T \boldsymbol{A}^T \boldsymbol{\Sigma}_{\boldsymbol{u}}^{(i)^{-1}} \boldsymbol{A} \bar{\boldsymbol{M}}\right)^{-1} \bar{\boldsymbol{M}}^T \boldsymbol{A}^T \boldsymbol{\Sigma}_{\boldsymbol{u}}^{(i)^{-1}} \boldsymbol{p}^{(i)} . \tag{15}$$

Least Squares Algorithm

Algorithm 2 `The LS Algorithm`

1: Input time-stamps $\{S_{j,k}, R_{j,k}, \bar{S}_{j,k}, \bar{R}_{j,k}\}_{k=1}^{K}$ and anchor locations
2: Initialize $\hat{x}^{(0)}$.
3: **for** $k = 1, \ldots, K$ **do**
4: Determine the MMSE estimator $\hat{\theta}_{k|K}^{(i)}$ from the Kalman smoother.
5: **end for**
6: Obtain the LS estimate $\hat{x}^{(i+1)}$ using (15).
7: **return**

- The LS algorithm presents a simple closed form alternative to the 2-D norm minimization in the M-step.
- Performance is expected to be similar to EM for low noise variance.

HCRB

- The covariance matrix of $\hat{\xi} = \left[\hat{\Theta}^T, x^T\right]^T$ is lower bounded as follows

Theorem

$$\mathbb{E}\left[\left(\hat{\xi} - \xi\right)\left(\hat{\xi} - \xi\right)^T\right] \succeq [\mathcal{H}(\Theta, x)]^{-1} \ ,$$

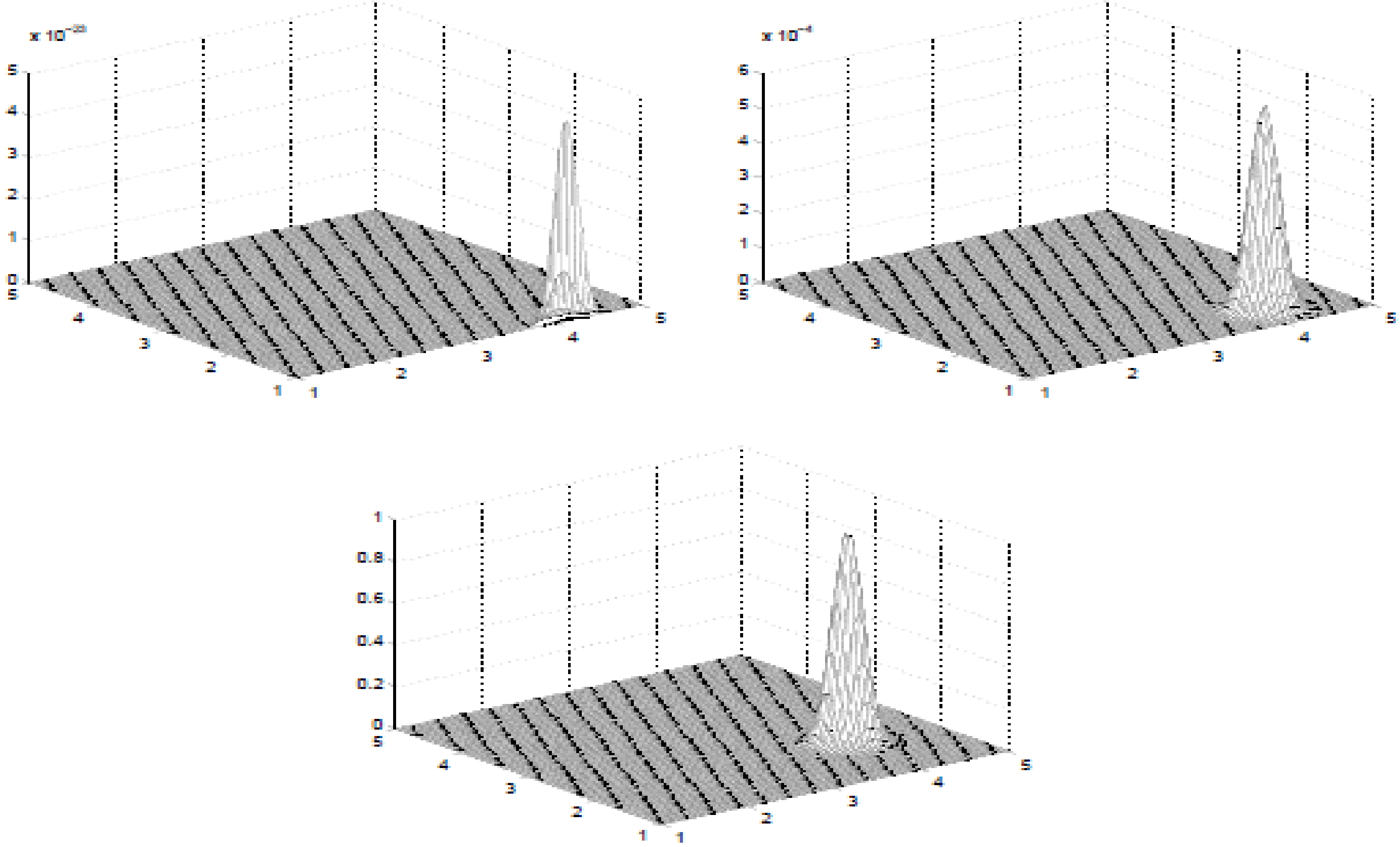
where the hybrid information matrix $\mathcal{H}(\Theta, x)$ *has entries*

$$\mathcal{H}_{11} = blkdiag\left(\frac{H_1^T H_1}{\sigma_w^2}, \ldots, \frac{H_K^T H_K}{\sigma_w^2}\right) + \Upsilon, \quad \mathcal{H}_{22} = \frac{2K}{\sigma_w^2} \sum_{j=1}^{N} \frac{(x - s_j)(x - s_j)^T}{\|x - s_j\|^2}$$

$$\mathcal{H}_{12} = \mathcal{H}_{21}^T = \left[\left(\frac{H_1^T d'(x)}{\sigma_w^2}\right)^T, \ldots, \left(\frac{H_K^T d'(x)}{\sigma_w^2}\right)^T\right]^T ,$$

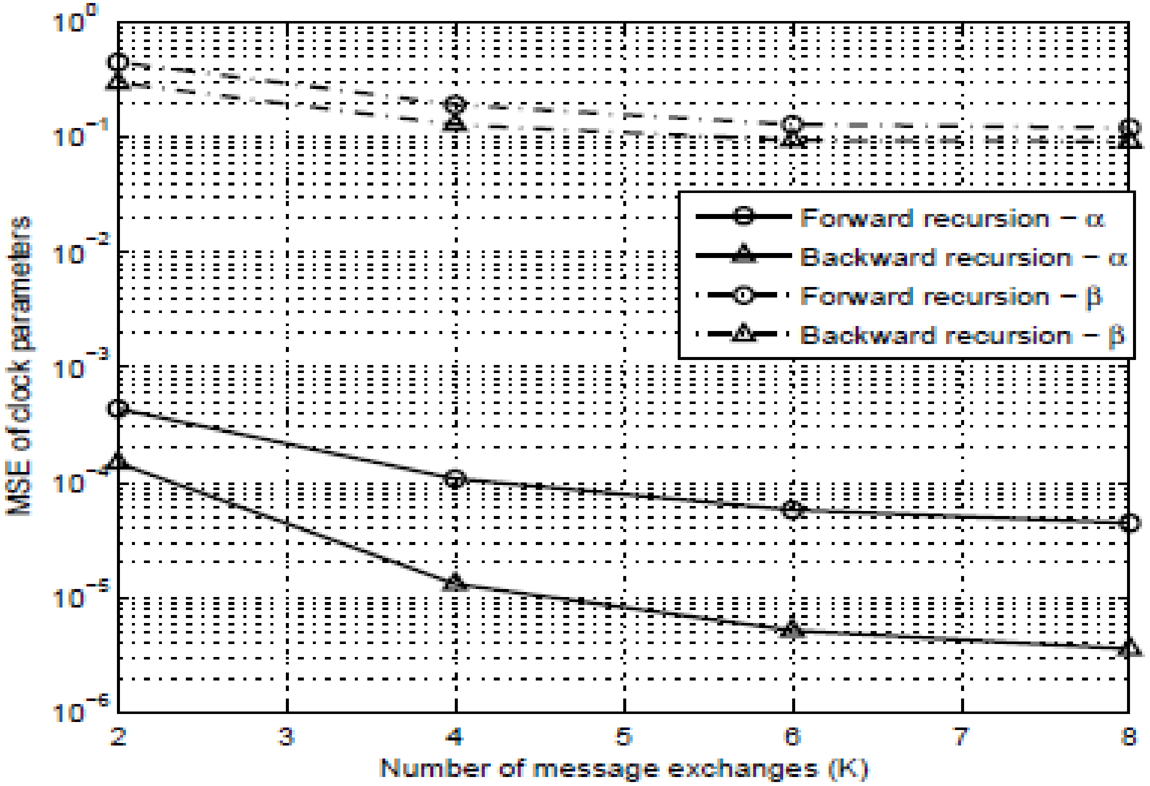
- The lower bound helps to benchmark the performance of EM and LS estimators.

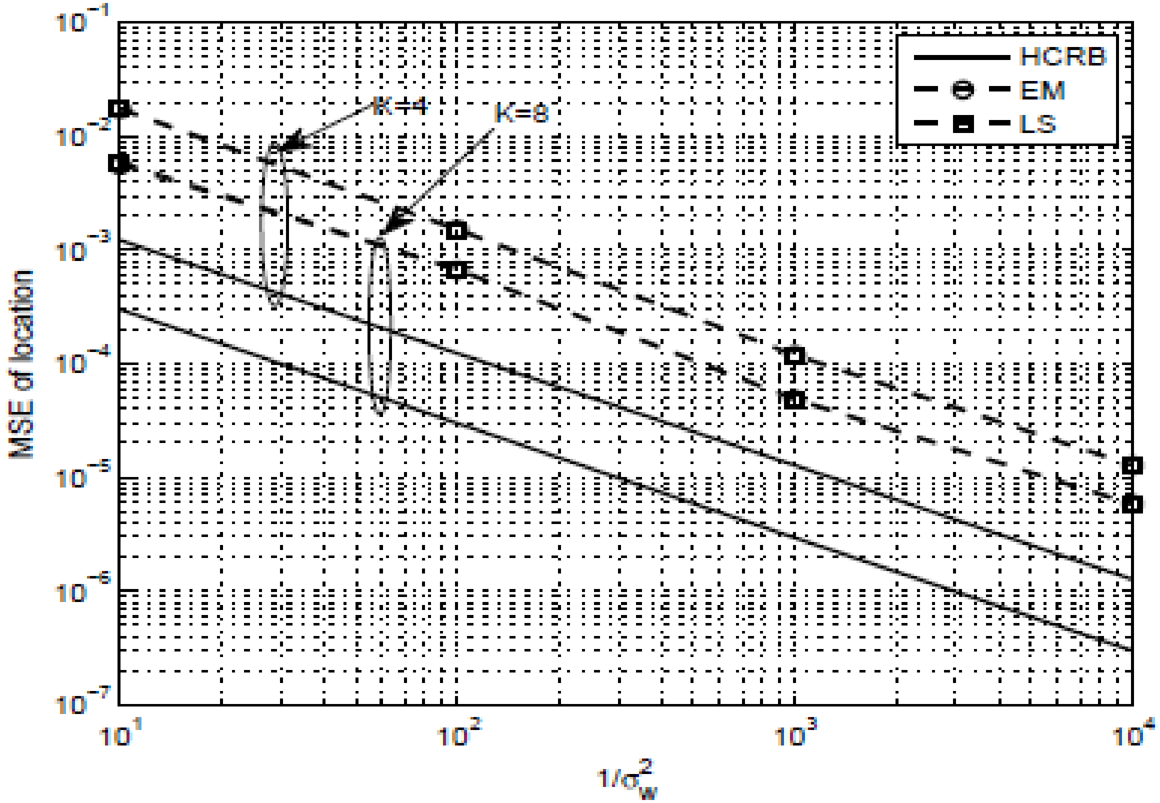
Simulation Results



- Fig. shows updates of $\exp\left(Q\left(x, \hat{x}^{(i)}\right)\right)$ for $x = (2, 4)$.
- We converge at the true coordinates in about $i = 12$ iterations.

Simulation Results





- The backward step yields an improvement in the estimates of clock parameters with increasing K.
- MSE of location estimates decrease with σ_w^2 but does not achieve the bound. Could be that the lower bound is unachievable (Messer, 2006).
- EM and LS estimators have similar performance for low to moderate noise variance.

Conclusions

- Convex optimization techniques used to provide alternative proofs of ML estimators derived by graphical maximization in prior contributions.
- A novel factor graph approach is proposed to incorporate the effects of time-variations in clock parameters.
- The results have been extended to network-wide clock synchronization in WSNs.
- Identifying the close connection between localization and synchronization, a joint estimation approach is proposed.
- Performance bounds have been derived to benchmark estimators. Can be useful in other problems in parameter estimation theory!
- Our work also demonstrates the potential of graphical models to solve inference problems in wireless communications.

Future Work

- Develop synchronization algorithms that do not assume a specific distribution of the network delays.
- Incorporate the effect of clock skew for network-wide synchronization. This can reduce re-synchronization requests.
- Incorporate node mobility and location uncertainty in joint localization and synchronization
    - Random walk model
    - Random way point model
    - Position, velocity, acceleration model
- Holy Grail! A mathematical proof of convergence of loopy belief propagation in general graphs.

Thank you!

Questions?