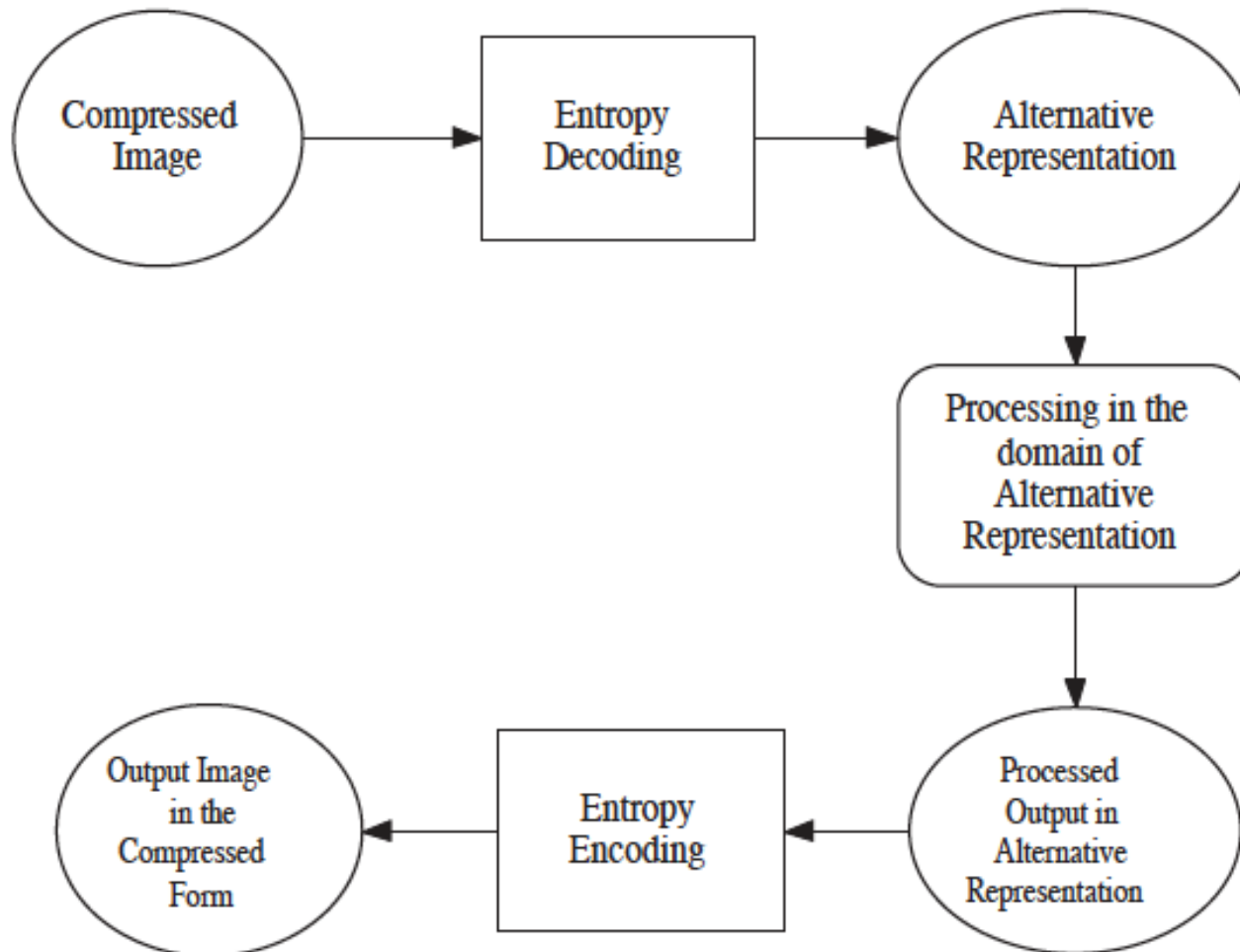


Color image processing in the compressed domain



Jayanta Mukhopadhyay
Dept. of Computer Science and Engg.
Indian Institute of Technology, Kharagpur

Processing with compressed image: Compressed domain approach



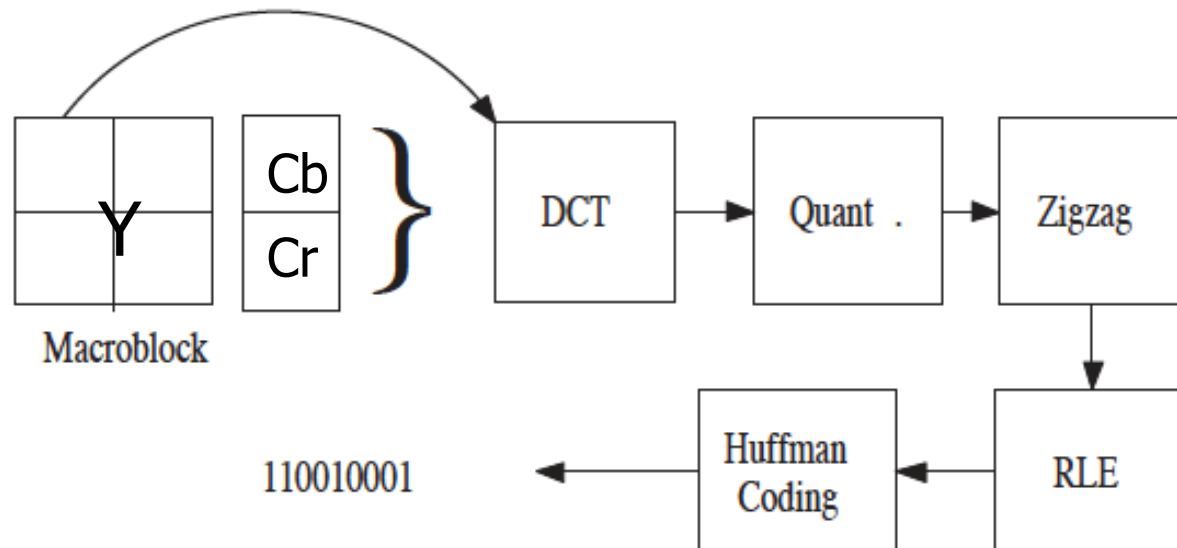
Color encoding in JPEG

- Y-Cb-Cr color space:

$$Y = 0.502G + 0.098B + 0.256R,$$

$$Cb = -0.290G + 0.438B - 0.148R + 128,$$

$$Cr = -0.366G - 0.071B + 0.438R + 128.$$





Motivations

- Computation with reduced storage.
- Avoid overhead of forward and reverse transform.
- Exploit spectral factorization for improving the quality of result and speed of computation.
- Color Image Processing in DCT domain.
 - Filtering
 - Enhancement
 - Color constancy
 - Resizing



2D DCT

$$\alpha(k) = \begin{cases} \sqrt{\frac{1}{2}} & k = 0 \\ 1 & \text{otherwise} \end{cases} \quad \beta(k) = \begin{cases} \frac{1}{2} & k = 0, N \\ 1 & \text{Otherwise} \end{cases}$$

- Type-I Even:

$$C_N^I = \left[\sqrt{\frac{2}{N}} \beta(k) \cos\left(\frac{\pi kn}{N}\right) \right]_{0 \leq (k,n) \leq N}$$

- Type-II Even:

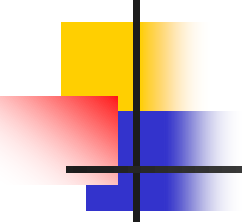
$$C_N^{II} = \left[\sqrt{\frac{2}{N}} \alpha(k) \cos\left(\frac{\pi k(2n+1)}{2N}\right) \right]_{0 \leq (k,n) \leq N-1}$$

- Type-I DCT of $h(m,n)$:

$$H = C_M^I h C_N^{IT}$$

- Type-II DCT of $x(m,n)$:

$$X = C_M^{II} x C_N^{IIT}$$



Useful properties of DCT blocks



2D DCT: Sub-band relation

$$x_{LL}(m, n) = \frac{1}{4} \{x(2m, 2n) + x(2m + 1, 2n) + x(2m, 2n + 1) + x(2m + 1, 2n + 1)\}, \quad 0 \leq m, n \leq \frac{N}{2} - 1.$$

Sub-band approximation:

2D DCT of $x_{LL}(m, n)$

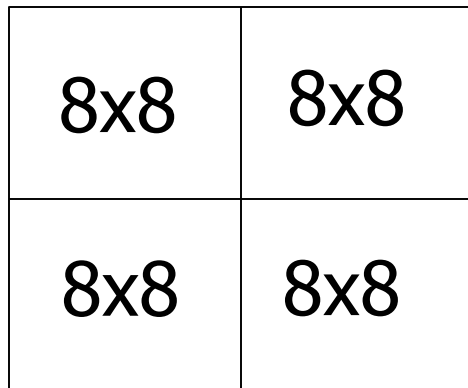
$$X(k, l) = \begin{cases} 2 \cos\left(\frac{\pi k}{2N}\right) \cos\left(\frac{\pi l}{2N}\right) \overline{X_{LL}}(k, l), & k, l = 0, 1, \dots, \frac{N}{2} - 1 \\ 0, & \text{otherwise.} \end{cases}$$

Low-pass truncated approximation:

$$X(k, l) = \begin{cases} 2 \overline{X_{LL}}(k, l), & k, l = 0, 1, \dots, \frac{N}{2} - 1 \\ 0, & \text{otherwise.} \end{cases}$$

S.-H. Jung, S.K. Mitra, and D. Mukherjee, Subband DCT: Definition, analysis and applications. IEEE Trans. on Circuits and systems for VideoTechnology, 6(3):273-286, June 1996.

Image downsampling



Sub-band approximation

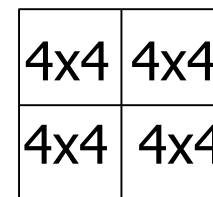
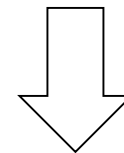
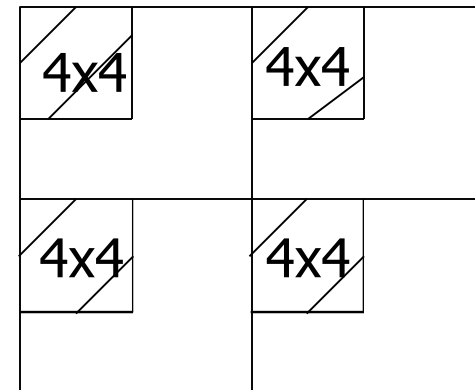
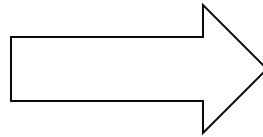
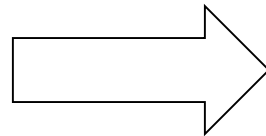


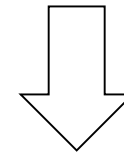
Image upsampling

4x4	4x4
4x4	4x4

Sub-band
approximation



4x4	0	4x4	0
0	0	0	0
4x4	0	4x4	0
0	0	0	0



8x8	8x8
8x8	8x8

2D DCT: Block composition and decomposition

$$X^{(LN \times MN)} = A_{(L,N)} \begin{bmatrix} X_{0,0}^{(N \times N)} & X_{0,1}^{(N \times N)} & \cdots & X_{0,M-1}^{(N \times N)} \\ X_{1,0}^{(N \times N)} & X_{1,1}^{(N \times N)} & \cdots & X_{1,M-1}^{(N \times N)} \\ \vdots & \vdots & \ddots & \vdots \\ X_{L-1,0}^{(N \times N)} & X_{L-1,1}^{(N \times N)} & \cdots & X_{L-1,M-1}^{(N \times N)} \end{bmatrix} A_{(M,N)}^T$$

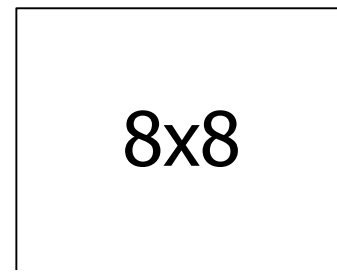
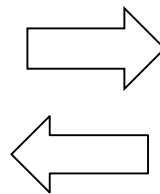
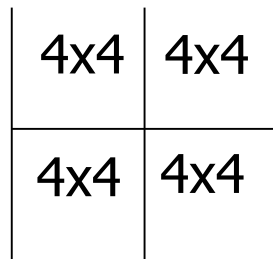
$$\begin{bmatrix} X_{0,0}^{(N \times N)} & X_{0,1}^{(N \times N)} & \cdots & X_{0,M-1}^{(N \times N)} \\ X_{1,0}^{(N \times N)} & X_{1,1}^{(N \times N)} & \cdots & X_{1,M-1}^{(N \times N)} \\ \vdots & \vdots & \ddots & \vdots \\ X_{L-1,0}^{(N \times N)} & X_{L-1,1}^{(N \times N)} & \cdots & X_{L-1,M-1}^{(N \times N)} \end{bmatrix} = A_{(L,N)}^{-1} X^{(LN \times MN)} A_{(M,N)}^{-1T}$$

Block composition and decomposition

$$A_{(2,4)} = C_8 \cdot \begin{bmatrix} C_4^{-1} & 0_4 \\ 0_4 & C_4^{-1} \end{bmatrix}$$

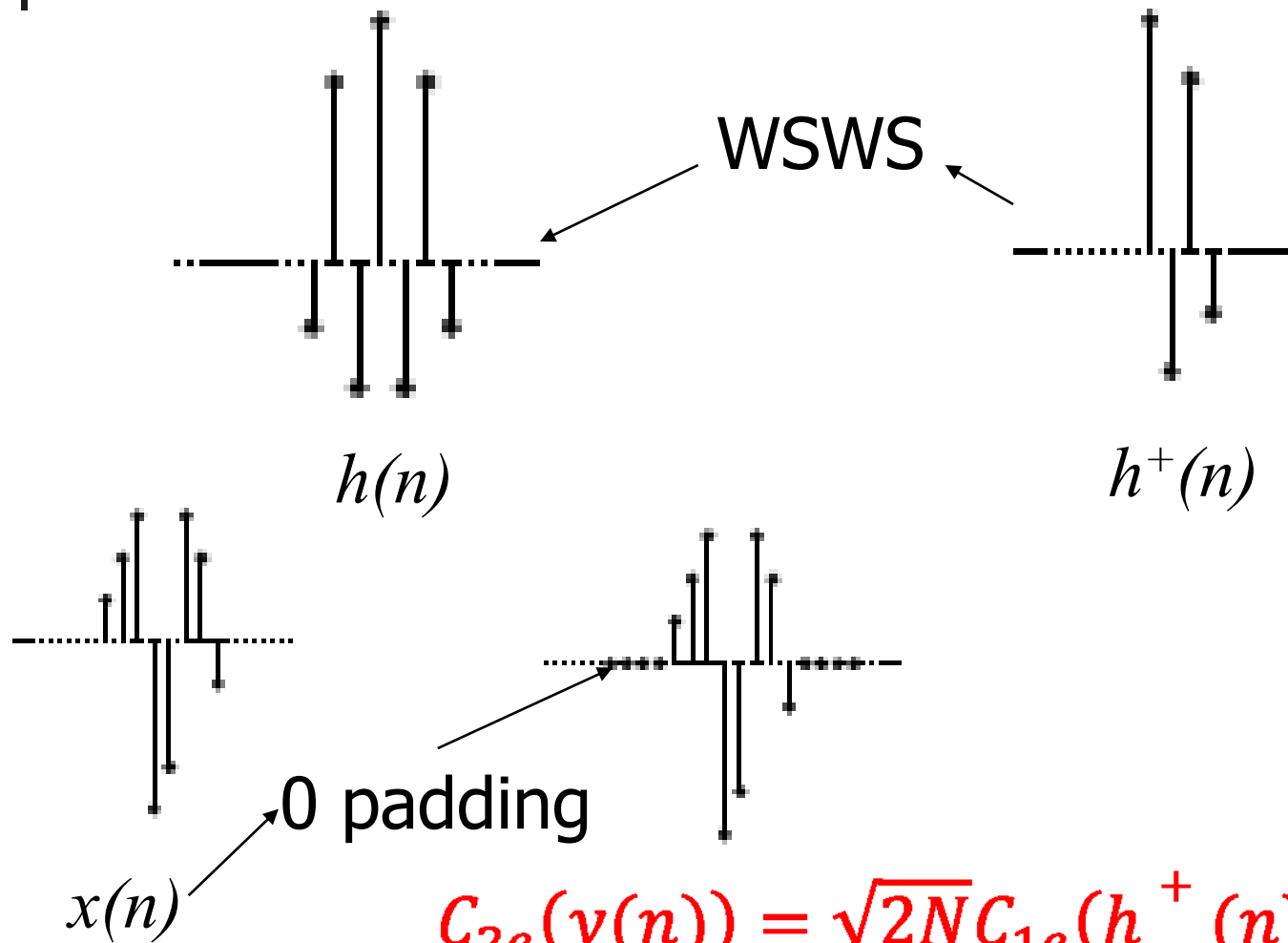
$$= \begin{bmatrix} 0.7071 & 0 & 0 & 0 & 0.7071 & 0 & 0 & 0 \\ 0.6407 & 0.294 & -0.0528 & 0.0162 & -0.6407 & 0.294 & 0.0528 & 0.0162 \\ 0 & 0.7071 & 0 & 0 & 0 & 0.7071 & 0 & 0 \\ -0.225 & 0.5594 & 0.3629 & -0.0690 & 0.225 & 0.5594 & -0.3629 & -0.069 \\ 0 & 0 & 0.7071 & 0 & 0 & 0 & 0.7071 & 0 \\ 0.1503 & -0.2492 & 0.5432 & 0.3468 & -0.1503 & -0.2492 & -0.5432 & 0.3468 \\ 0 & 0 & 0 & 0.7071 & 0 & 0 & 0 & -0.7071 \\ -0.1274 & 0.1964 & -0.2654 & 0.6122 & 0.1274 & 0.1964 & 0.2654 & 0.6122 \end{bmatrix}$$

$$X^{(8 \times 8)} = A_{(2,4)} \begin{bmatrix} X_{00}^{(4 \times 4)} & X_{01}^{(4 \times 4)} \\ X_{10}^{(4 \times 4)} & X_{11}^{(4 \times 4)} \end{bmatrix} A_{(2,4)}^T$$



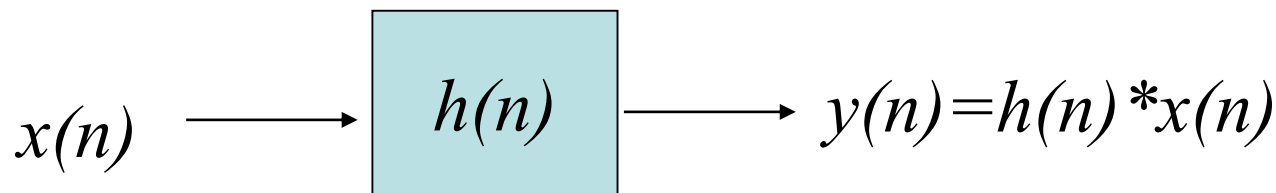
$$\begin{bmatrix} X_{00}^{(4 \times 4)} & X_{01}^{(4 \times 4)} \\ X_{10}^{(4 \times 4)} & X_{11}^{(4 \times 4)} \end{bmatrix} = A_{(2,4)}^{-1} X^{(8 \times 8)} A_{(2,4)}^{-T}$$

Symmetric convolution and CMP for type II DCT

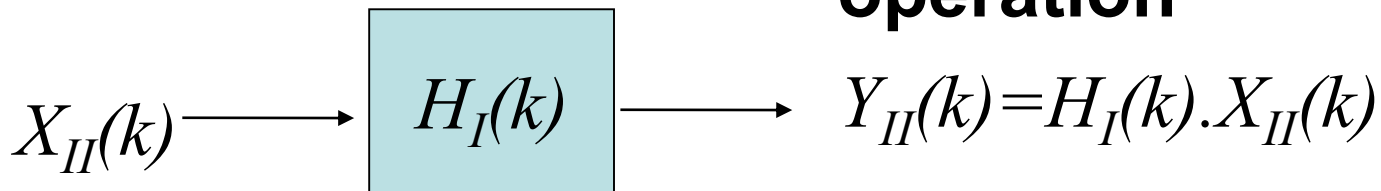


$$C_{2e}(y(n)) = \sqrt{2N} C_{1e}(h^+(n)) C_{2e}(x(n))$$

CONVOLUTION- MULTIPLICATION PROPERTY



\Updownarrow • **Symmetric convolution operation**



$$Y_{II}^{(N)} = \begin{bmatrix} H_{I,0} & 0 & 0 & 0 \\ 0 & H_{I,1} & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & H_{I,N-1} \end{bmatrix} X_{II}^{(N)}$$

$\longleftarrow \text{diag}(H_I)$

Diagonal matrix formed by $H_I(k)$



2D DCT: CMPs

Circular convolution with
respective symmetric extensions.

$$C_{2e}\{x(m, n) \circledast h(m, n)\} = C_{2e}\{x(m, n)\} C_{1e}\{h(m, n)\}$$

$$C_{1e}\{x(m, n) \circledast h(m, n)\} = C_{2e}\{x(m, n)\} C_{2e}\{h(m, n)\}$$

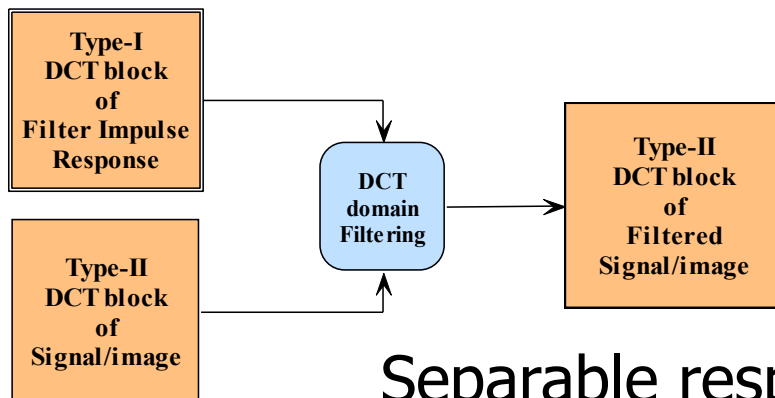


DCT Domain Filtering

Filtering in 2-D block DCT space

Given, **type-I DCT** of the impulse response of a filter and an input in **type-II DCT space**, filtered output can be transformed in the same space (i.e. type-II DCT space).

$$\begin{aligned} Y_{II}^{(N \times N)} &= \left(\text{diag}(H_I) \left(\text{diag}(H_I) X_{II}^{(N \times N)} \right)^T \right)^T \\ &= \text{diag}(H_I) X_{II}^{(N \times N)} \text{diag}(H_I)^T \\ &= \text{diag}(H_I) X_{II}^{(N \times N)} \text{diag}(H_I) \end{aligned}$$



Separable response: $h(x,y)=h(x)h(y)$

BOUNDARY EFFECT IN BLOCK DCT DOMAIN

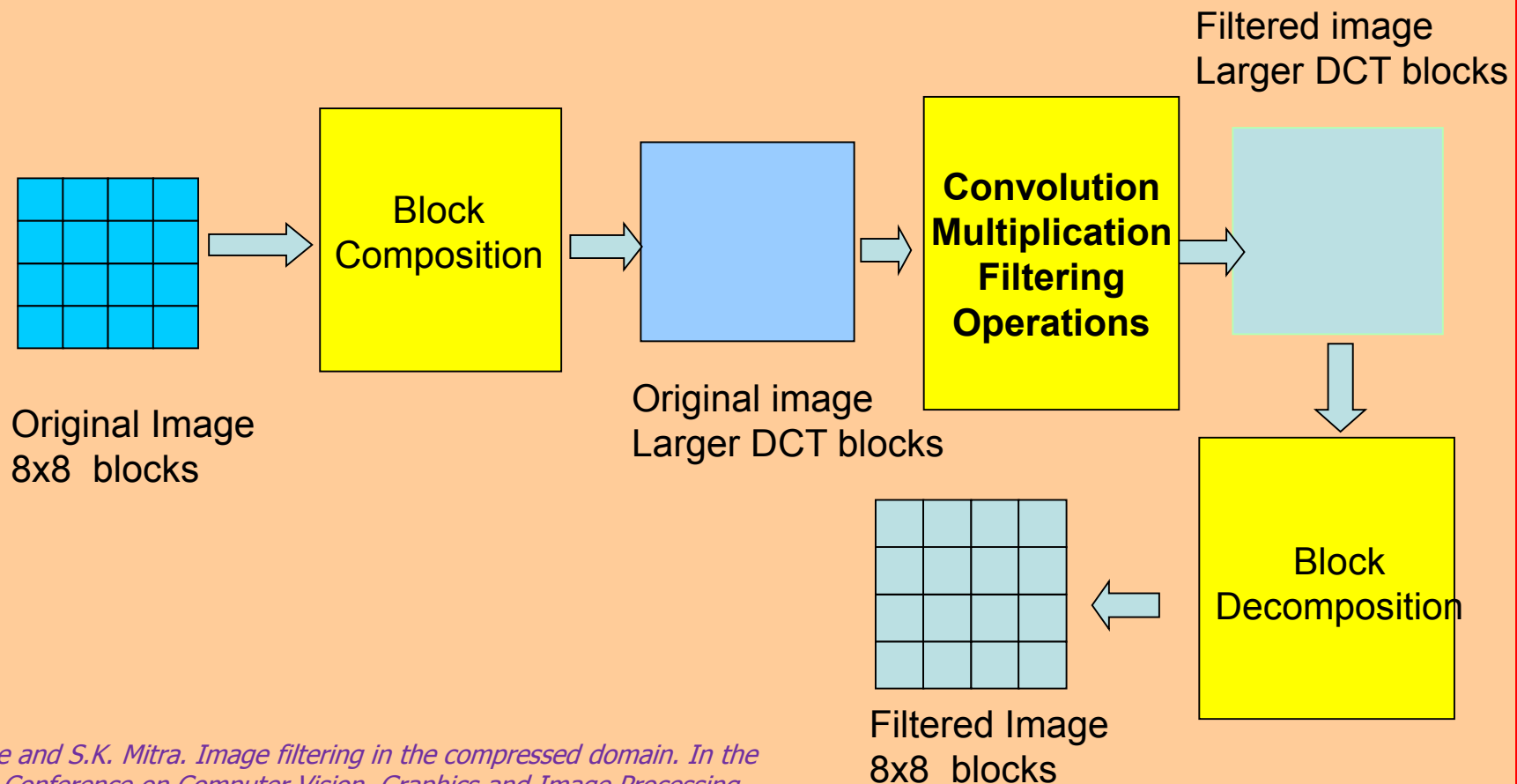


Linear Convolution



**Block Symmetric
Convolution**

Computation with larger blocks: The BFCD filtering Algorithm





Composite operation

- Block composition + Multiplication with filter coefficients + Block decomposition can be expressed by a composite matrix for the linear operation.

In 1-D: For filtering 3 NxN adjacent DCT blocks:

$$U^{(3N \times 3N)} = A_{(3,N)}^T \mathbb{D}(\{\sqrt{6N} C_{3N}^T \mathbf{h}^+\}_{0}^{3N-1}) A_{(3,N)}$$

$$\begin{bmatrix} Y_1^{(N)} \\ Y_2^{(N)} \\ Y_3^{(N)} \end{bmatrix} = U \begin{bmatrix} X_1^{(N)} \\ X_2^{(N)} \\ X_3^{(N)} \end{bmatrix}$$

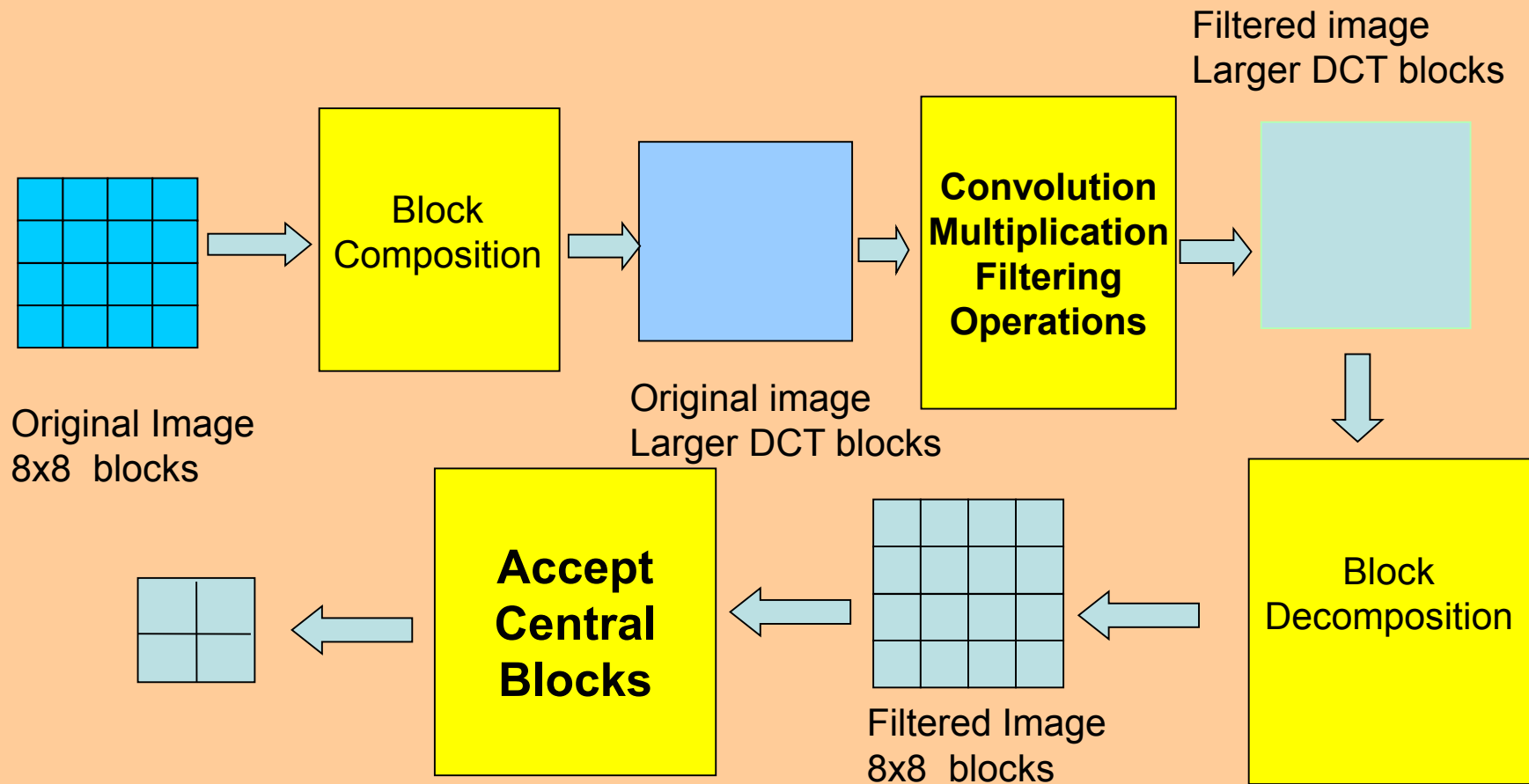
Decomp.
Multiplication
Composition



In 2-D

$$\begin{bmatrix} Y_{11}^{(N \times N)} & Y_{12}^{(N \times N)} & Y_{13}^{(N \times N)} \\ Y_{21}^{(N \times N)} & Y_{22}^{(N \times N)} & Y_{23}^{(N \times N)} \\ Y_{31}^{(N \times N)} & Y_{32}^{(N \times N)} & Y_{33}^{(N \times N)} \end{bmatrix} = U \begin{bmatrix} X_{11}^{(N \times N)} & X_{12}^{(N \times N)} & X_{13}^{(N \times N)} \\ X_{21}^{(N \times N)} & X_{22}^{(N \times N)} & X_{23}^{(N \times N)} \\ X_{31}^{(N \times N)} & X_{32}^{(N \times N)} & X_{33}^{(N \times N)} \end{bmatrix} U^T$$

Exact Computation: The Overlapping and Save (OBFCD) filtering Algorithm



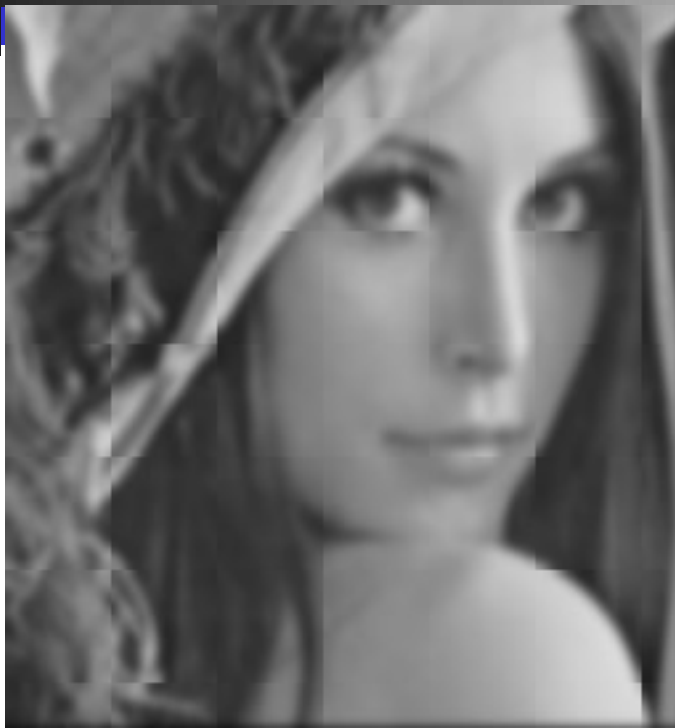


Overlap and Save Strategy

$$\begin{bmatrix} Y_{11}^{(N \times N)} & Y_{12}^{(N \times N)} & Y_{13}^{(N \times N)} \\ Y_{21}^{(N \times N)} & Y_{22}^{(N \times N)} & Y_{23}^{(N \times N)} \\ Y_{31}^{(N \times N)} & Y_{32}^{(N \times N)} & Y_{33}^{(N \times N)} \end{bmatrix} = U \begin{bmatrix} X_{11}^{(N \times N)} & X_{12}^{(N \times N)} & X_{13}^{(N \times N)} \\ X_{21}^{(N \times N)} & X_{22}^{(N \times N)} & X_{23}^{(N \times N)} \\ X_{31}^{(N \times N)} & X_{32}^{(N \times N)} & X_{33}^{(N \times N)} \end{bmatrix} U^T$$

Save only the central block.

Filtered Images



BFCD Algorithm (5x5)



OBFCD Algorithm

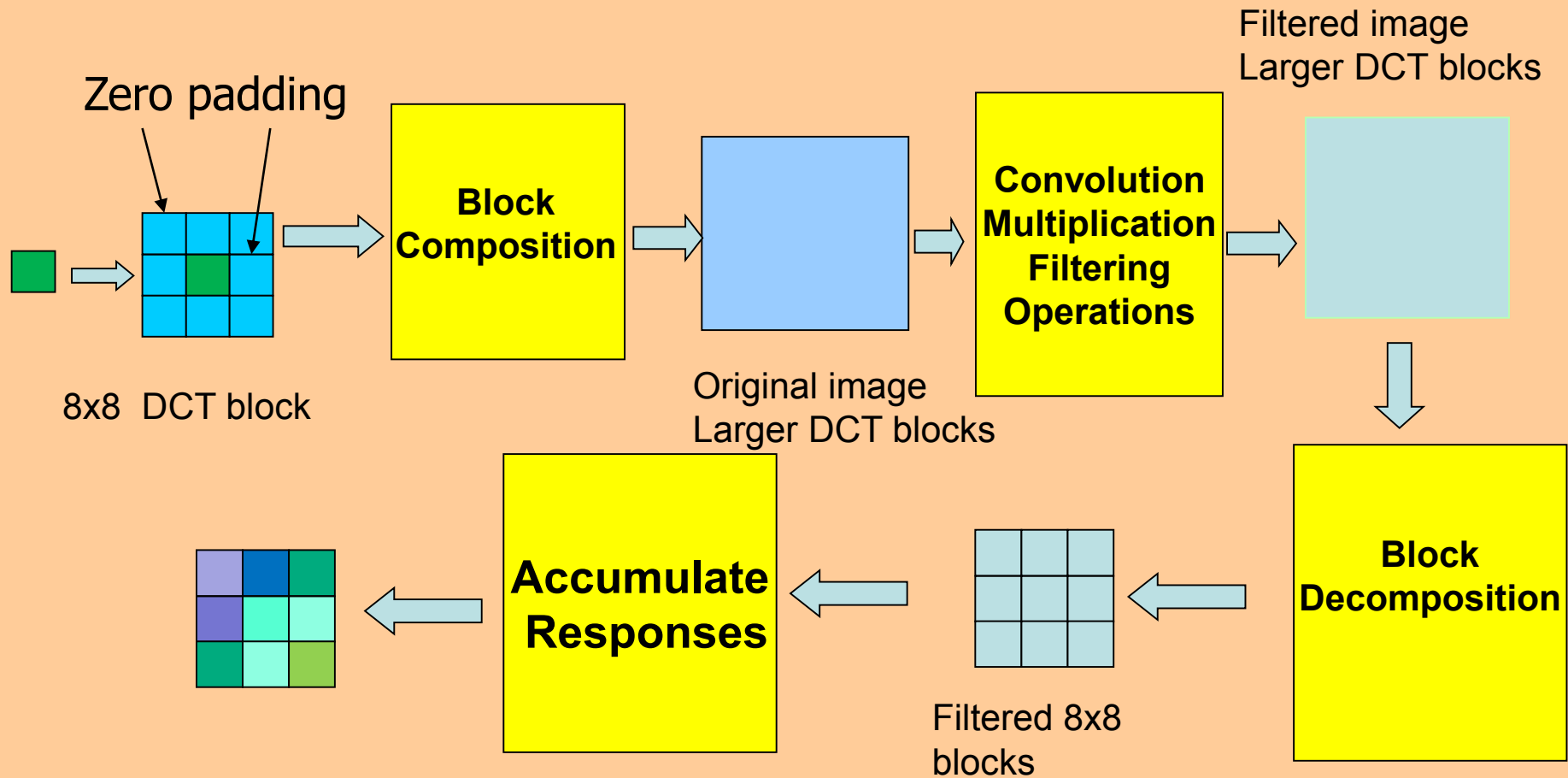


Overlap and add strategy

$$\begin{bmatrix} Y_{11}^{(N \times N)} & Y_{12}^{(N \times N)} & Y_{13}^{(N \times N)} \\ Y_{21}^{(N \times N)} & Y_{22}^{(N \times N)} & Y_{23}^{(N \times N)} \\ Y_{31}^{(N \times N)} & Y_{32}^{(N \times N)} & Y_{33}^{(N \times N)} \end{bmatrix} = U \begin{bmatrix} X_{11}^{(N \times N)} & X_{12}^{(N \times N)} & X_{13}^{(N \times N)} \\ X_{21}^{(N \times N)} & X_{22}^{(N \times N)} & X_{23}^{(N \times N)} \\ X_{31}^{(N \times N)} & X_{32}^{(N \times N)} & X_{33}^{(N \times N)} \end{bmatrix} U^T$$

- For computing contribution of the central block X_{22} to neighboring blocks all other blocks in the input set to zeros.
- Add accumulated contribution at every block.

Exact Computation: The Overlapping and Add filtering Algorithm



Removal of Blocking Artifacts



**Blocking artifacts
Compressed with quality
factor 10**



**Artifacts removed
By filtering in DCT
domain**

Image Sharpening in DCT domain

$$X_s = X + k \cdot (X - X_{lpf})$$



Lena



Peppers

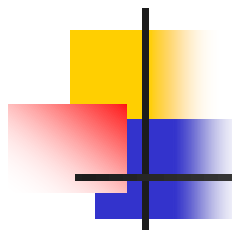


Color preservation in the compressed domain

$$\begin{aligned} Y &= 0.502G + 0.098B + 0.256R, \\ Cb &= -0.290G + 0.438B - 0.148R + 128, \\ Cr &= -0.366G - 0.071B + 0.438R + 128. \end{aligned}$$

- R-G-B to Y-Cb-Cr : Affine
- R-G-B to Y-Cb'-Cr', where Cb'=Cb-128 & Cr'=Cr-128: Linear
- DC of a Y-block (Y_{dc}) scaled by a factor k.
- Corresponding Cb'_{dc} and Cr'_{dc} should also be scaled by a factor k for color preservation.
- Transform Cb'_{dc} and Cr'_{dc} to Cb_{dc} and Cr_{dc} then by simple addition.
 - $Cb_{dc} = Cb'_{dc} + 128N$ $Cr_{dc} = Cr'_{dc} + 128N$

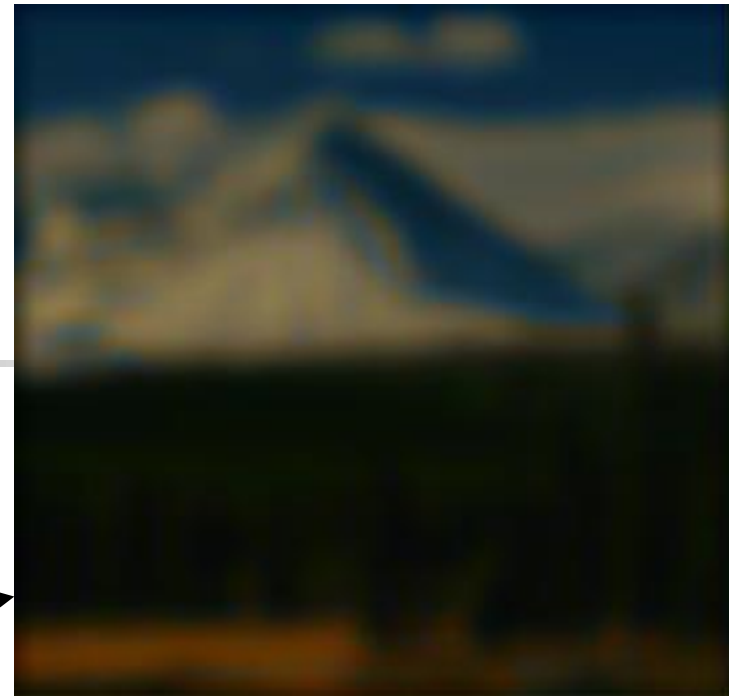
Color Sharpening



Original



Filtered



Sharpened →





Color Saturation and Desaturation

CIE chromaticity chart

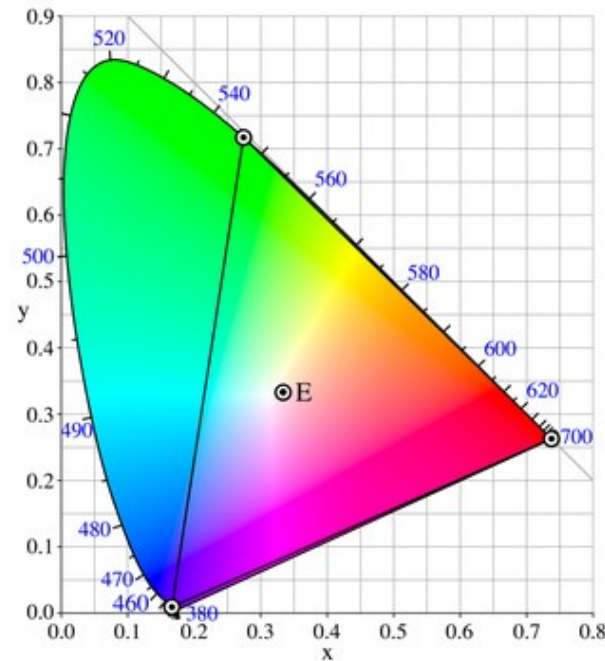
- The Commission Internationale de l'Éclairage (estd. 1931) defined 3 hypothetical additive primaries: X, Y, Z

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 0.6067 & 0.1736 & 0.2001 \\ 0.2988 & 0.5868 & 0.1143 \\ 0.0000 & 0.0661 & 1.1149 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

- Normalized x - y space:

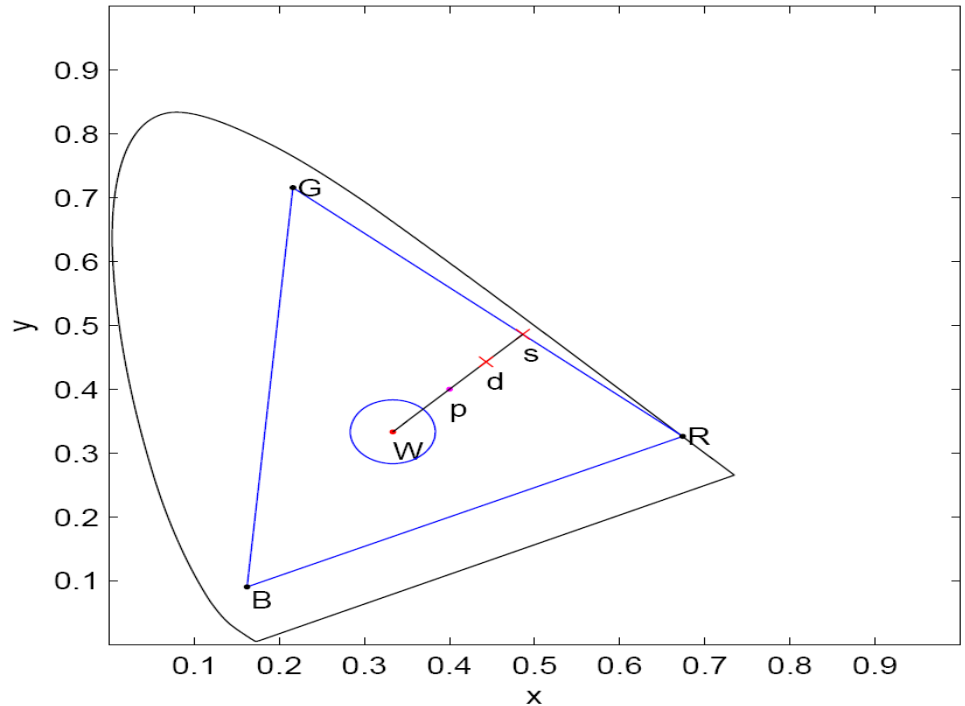
$$x = X/(X+Y+Z)$$

$$y = Y/(X+Y+Z)$$



Saturation and De-saturation Operation

- Move radially to the gamut edge \rightarrow Maximum Saturation given a hue.
- Move inward using center of gravity law of color mixing.



Desaturation using Center of Gravity Law

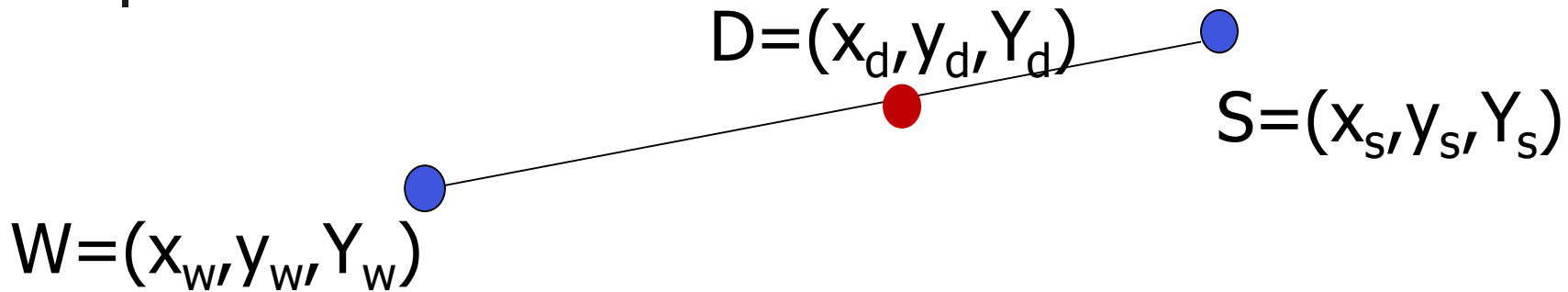
The *Center of Gravity Law* provides the resulting color $C_2 = (x_2, y_2, Y_2)$ of the mixture of the two colors $W = (x_w, y_w, |Y_w|)$ and $S = (x_s, y_s, Y_1)$ where

$$x_2 = \frac{x_w \frac{|Y_w|}{y_w} + x_s \frac{Y_1}{y_s}}{\frac{|Y_w|}{y_w} + \frac{Y_1}{y_s}}, \quad y_2 = \frac{|Y_w| + Y_1}{\frac{|Y_w|}{y_w} + \frac{Y_1}{y_s}}, \quad \text{and} \quad Y_2 = Y_w + Y_1. \quad (3)$$

$$Y_w = \kappa Y_{avg}$$

Apparent masses for chromatic mixture: Y_w/y_w and Y_1/y_s

Desaturation using Center of Gravity Law



$$x_d = \frac{x_w \frac{|Y_w|}{y_w} + x_s \frac{|Y_s|}{y_s}}{\frac{|Y_w|}{y_w} + \frac{|Y_s|}{y_s}}$$

$$y_d = \frac{|Y_w| + Y_s}{\frac{|Y_w|}{y_w} + \frac{|Y_s|}{y_s}}$$

$$Y_d = |Y_w| + Y_s$$

$$Y_w = kY_{avg}$$



nCb-nCr color space

$$nCb = \frac{Cb - 128}{aY + b(Cb - 128) + c(Cr - 128)}$$

$$a=3.51$$

$$b=1.99$$

$$nCr = \frac{Cr - 128}{aY + b(Cb - 128) + c(Cr - 128)}$$

$$c=0.14$$

$$\begin{bmatrix} nCb \\ nCr \end{bmatrix} = \begin{bmatrix} -0.6823 & -0.7724 \\ 1.532 & -0.6047 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 0.4849 \\ -0.3091 \end{bmatrix}$$

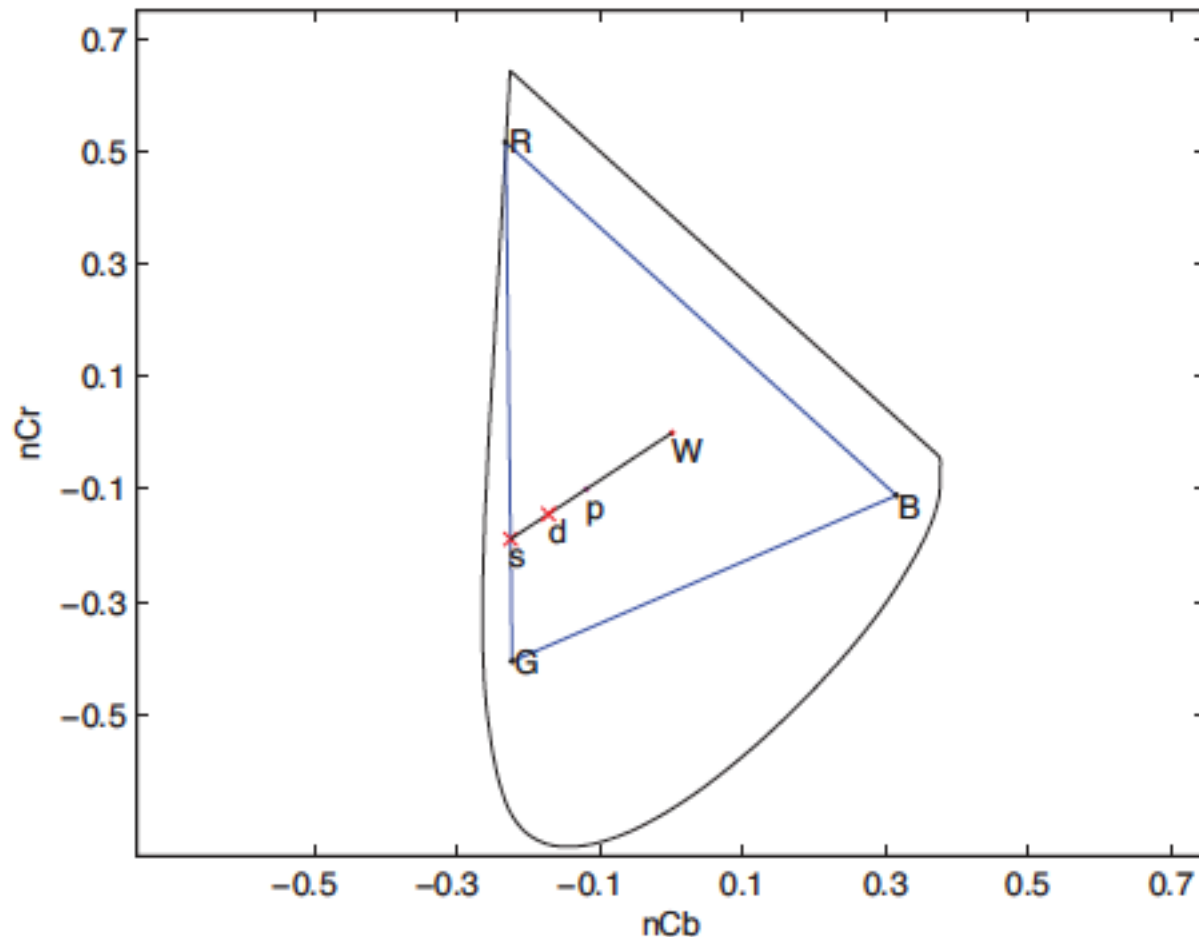
$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -0.3789 & 0.484 \\ -0.96 & -0.4275 \end{bmatrix} \begin{bmatrix} nCb \\ nCr \end{bmatrix} + \begin{bmatrix} \frac{1}{3} \\ \frac{1}{3} \end{bmatrix}$$



Cb-Cr from nCb-nCr given Y

$$\begin{bmatrix} Cb_s \\ Cr_s \end{bmatrix} = a.Y \begin{bmatrix} 1 - b.nCb & -c.nCb \\ -b.nCr & 1 - c.nCr \end{bmatrix}^{-1} \begin{bmatrix} nCb \\ nCr \end{bmatrix} + \begin{bmatrix} 128 \\ 128 \end{bmatrix}.$$

Saturation and De-saturation Operation in nCbCr space





Chromatic Shift in Cb-Cr space

- The ratio of (Cb-128) and (Cr-128) preserves the hue.
- Shift the (Cb,Cr) point along the line joining from (128,128) to it.
- Equivalent to saturation and desaturation operation.

original



Max-sat



Sat-desat



Chromatic-shift



original



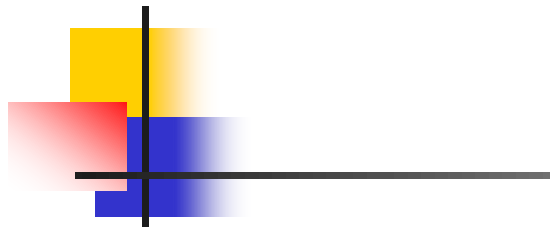
Max-sat

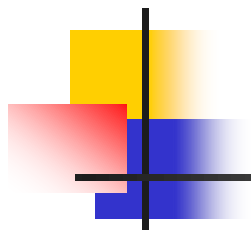


Sat-desat



Chromatic-shift





Color Enhancement



Contrast : Definition

Weber Law: $\zeta = \frac{\Delta L}{L}$

where ΔL is the difference in luminance between a stimulus and its surround, and L is the luminance of the surround

μ : mean of a block \leftarrow DC coefficient of the block

σ : standard deviation of a block \leftarrow Sum of AC coefficients

Contrast ζ of an image is defined here as: .

$$\zeta = \frac{\sigma}{\mu}$$

Theorem on Contrast Preservation in the DCT Domain

κ_d : the scale factor for the DC coefficient

κ_a : the scale factor for the AC coefficients

$$Y_e(i, j) = \begin{cases} \kappa_d Y(i, j), & i=j=0 \\ \kappa_a Y(i, j), & \text{otherwise} \end{cases}$$

The contrast of the processed image : κ_a / κ_d times of the contrast of the original image.

$\kappa_d = \kappa_a = \kappa$ preserves the contrast.

Preservation of Colors in the DCT Domain

U, V : Blocks of DCT coefficients of C_b and C_r
 κ : Scale factor for the luminance component Y

$$U_e(i, j) = \begin{cases} N(\kappa (\frac{U(i, j)}{N} - 128)) + 128, & i=j=0 \\ \kappa U(i, j), & \text{otherwise} \end{cases}$$

$$V_e(i, j) = \begin{cases} N(\kappa (\frac{V(i, j)}{N} - 128)) + 128, & i=j=0 \\ \kappa V(i, j), & \text{otherwise} \end{cases}$$

Color Enhancement by Scaling Coefficients (CES)

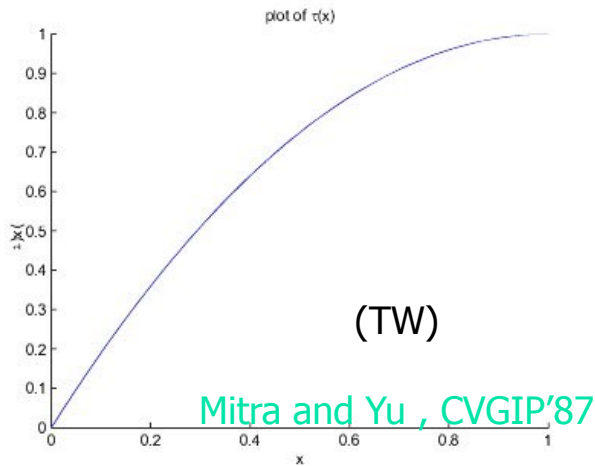
- Find the scale factor by mapping the DC coefficient with a monotonically increasing function.

$$1 \leq \kappa \leq \frac{B_{max}}{\mu + \lambda \cdot \sigma}$$

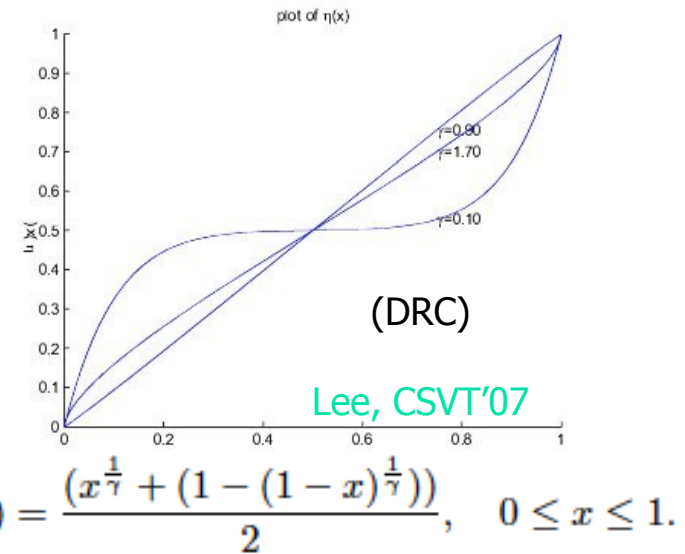
Scale factor κ ← Max. intensity

- Apply scaling to all other coefficients in all the components.
- For blocks having greater details judged by s.d., apply block decomposition and re-composition strategy.

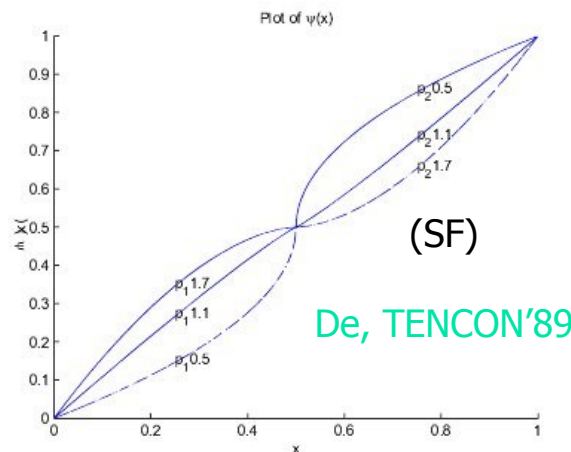
Mapping functions for adjusting the local background illumination



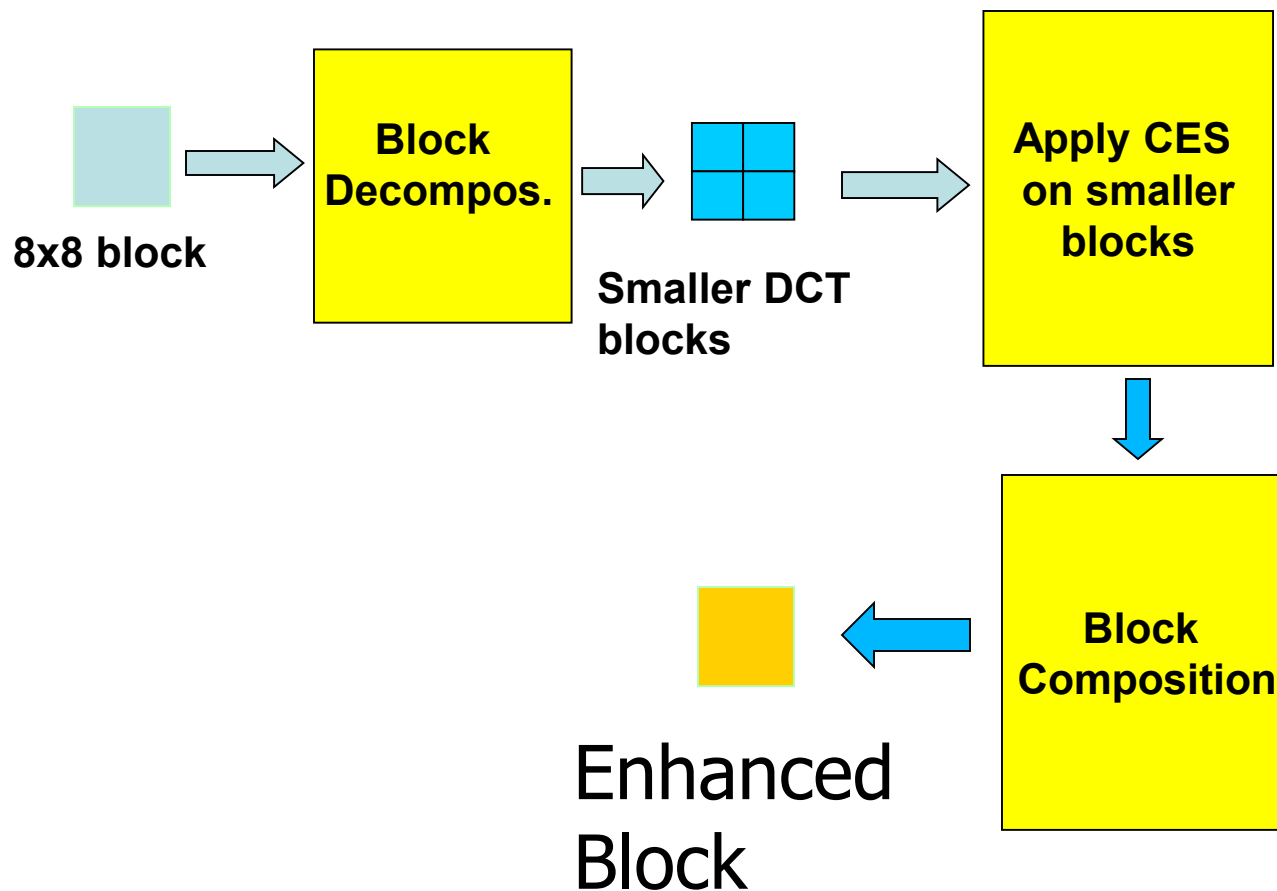
$$\kappa = \frac{f(Y(0,0))}{Y(0,0)}$$



$$\tau(x) = x(2 - x)$$



Enhancement of Blocks with more details







MCE



MCEDRC



CES

Some Results



original



AR



MCE



MCEDRC



TW-CES-BLK



MSR

Iterative Enhancement



Iteration no.=1



original



Iteration no.=3



Iteration no.=2



Iteration no.=4



0



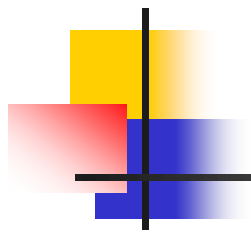
1



2



3



Color Constancy



Color constancy

- Computation of color of illuminant
 - Avg. of colors (Gray world)
 - Maximum of each channel (White world)
 - Use of DC coefficient of DCT blocks in compressed domain.

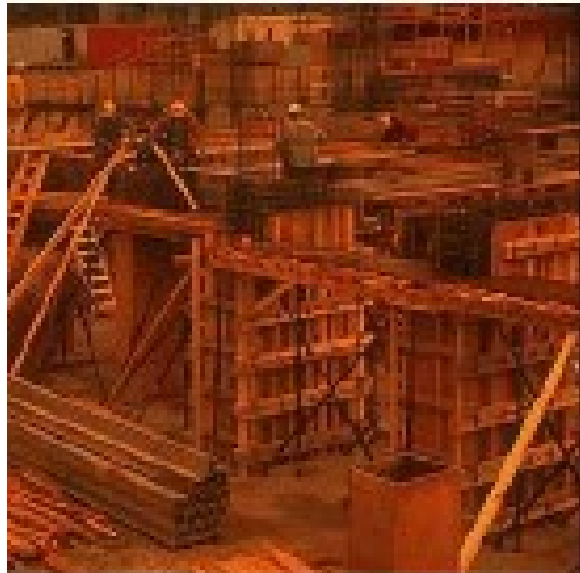
- Diagonal Color correction: $(R_s, G_s, B_s) \rightarrow (R_d, G_d, B_d)$

$$\begin{aligned}
 k_r &= \frac{R_d}{R_s}, & k_g &= \frac{G_d}{G_s}, & k_b &= \frac{B_d}{B_s}, \\
 f &= \frac{R+G+B}{k_r R + k_g G + k_b B}, \\
 R_u &= f k_r R, & G_u &= f k_g G, & B_u &= f k_b B.
 \end{aligned}$$

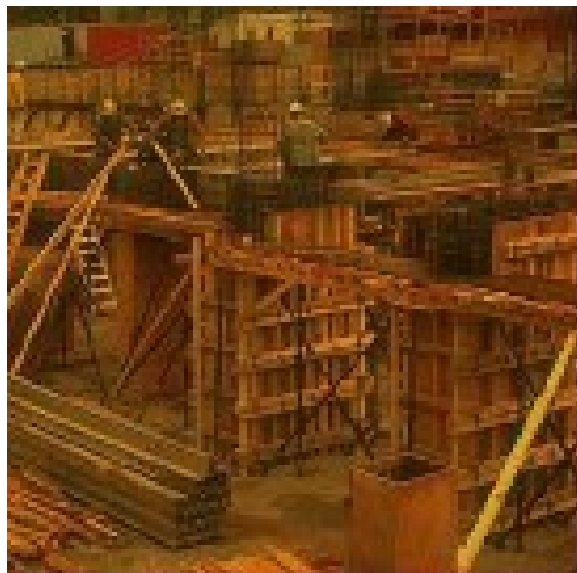
- Chromatic Shift in Y-Cb-Cr:

$$\begin{aligned}
 Y_u &= Y, \\
 C_{bu} &= C_b + C_{bd} - C_{bs}, \\
 C_{ru} &= C_r + C_{rd} - C_{rs}
 \end{aligned}$$

Color correction: Example



Original



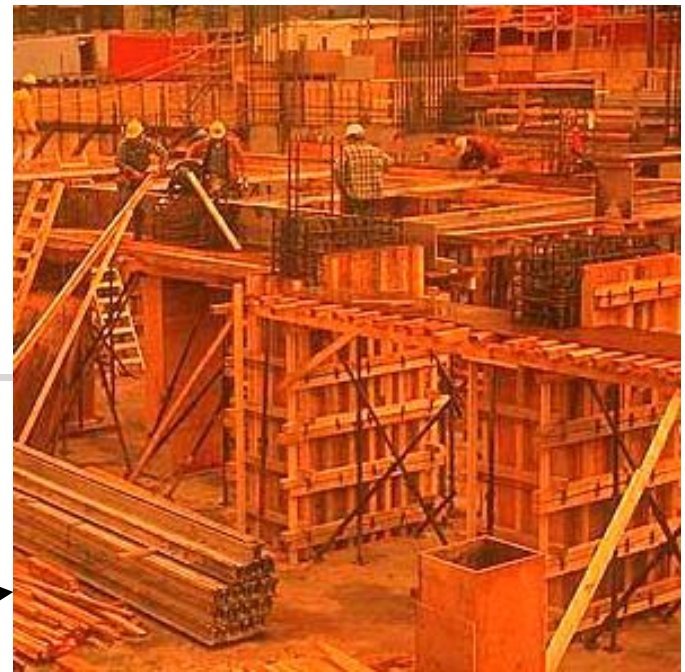
Diagonal Correction



Chromatic Shift

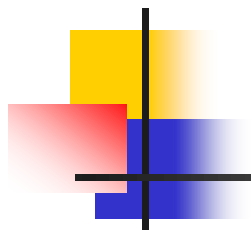
Color constancy coupled enhancement

- Perform color correction
- Perform color enhancement



Enhancement without color correction





Color Image Resizing

Resizing with integral factors

To convert $N \times N$ block to $LN \times MN$ block.

$LN \times MN$ block

$N \times N$ DCT block

$$X(k, l) = \begin{cases} \sqrt{LM} X_{LL}(k, l), & k, l = 0, 1, \dots, N-1, \\ 0, & \text{otherwise.} \end{cases}$$

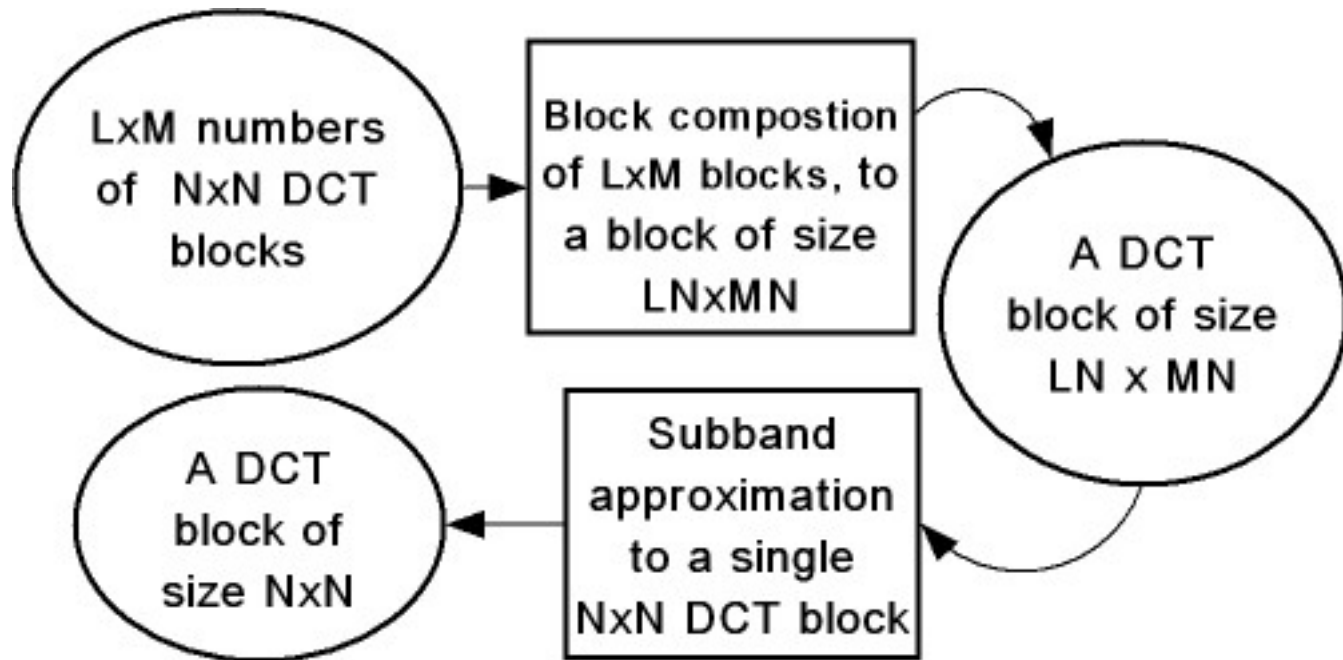
$L \times M$ D/S (LMDS) 1. Merge $L \times M$ adjacent DCT blocks.

$$Z = A_{(L,N)} \begin{bmatrix} X_{0,0} & X_{0,1} & \cdots & X_{0,(M-1)} \\ X_{1,0} & X_{1,1} & \cdots & X_{1,(M-1)} \\ \vdots & \vdots & \cdots & \vdots \\ X_{(L-1),0} & X_{(L-1),1} & \cdots & X_{(L-1),(M-1)} \end{bmatrix} A_{(M,N)}^T$$

2. Sub-band approximation to a $N \times N$ DCT block.

$$Y = \sqrt{\frac{1}{LM}} [Z(k, l)]_{0 \leq k, l \leq N-1}$$

LMDS





LxM U/S (LMUS)

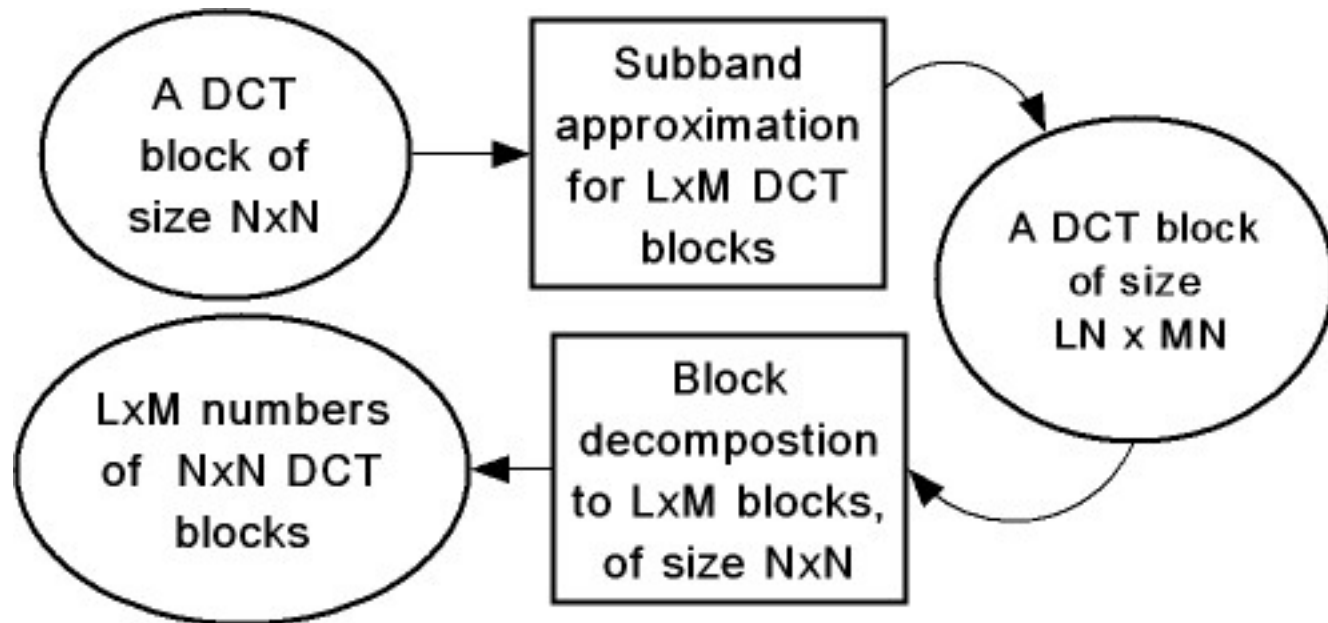
1. Convert NxN to LNxMN block

$$\hat{B} = \begin{bmatrix} \sqrt{LMB} & \mathbf{0}_{(N, (M-1)N)} \\ \mathbf{0}_{((L-1)N, N)} & \mathbf{0}_{((L-1)N, (M-1)N)} \end{bmatrix}$$

Efficiently
compute
exploiting large
blocks of
zeroes.

2. Decompose into LxM NxN blocks.

LMUS



An example:

3x2 D/S

and U/S





Arbitrary Resizing ($P/R \times Q/S$)

- U/S-D/S Resizing Algorithm (UDRA)
 - U/S by $P \times Q$
 - D/S by $R \times S$
- D/S-U/S Resizing Algorithm (DURA)
 - D/S by $R \times S$
 - U/S $P \times Q$

HDTV (1080x920) to NTSC (480x640)



UDRA



DURA





Issues involved in color resizing

- **Baseline JPEG Compression:**
Usually the chromatic components Cb and Cr are at lower resolution than the Y component.
- Cascaded stages of down-sampling and up-sampling (the DURA algorithm) faces a problem of dimensionality mismatch.
- Appearance of color artifacts in boundary.

Lighthouse (original)



3/4 x 4/3 Resizing



DURA



UDRA



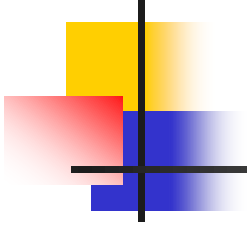
Conclusion (contd.)

- Useful properties of DCT exploited in developing algorithms.
 - Linear and distributive property.
 - Sub-band relationship
 - Spatial relationship
 - Convolution-Multiplication properties.
- Processing of DC coefficients adapts spatial domain algorithm in the DCT domain.
- Processing of AC coefficients also required to handle details, etc.



Conclusion

- Color space in compressed domain Y-Cb-Cr
 - Affine transformation from R-G-B space.
 - Usually Cb and Cr blocks are down-sampled.
- Processing in chromatic space required to preserve color vector, if luminance component (Y) is modified.



Thank you!