Technical University of Cluj-Napoca

Faculty of Electronics, Telecommunications and Information Technology

Basis of Electronics Department

RESEARCHES REGARDING PHASE APPROXIMATION FROM FREQUENCY RESPONSE GAIN SAMPLES

PhD Thesis Summary

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Hilbert transform and the related Bode relationships have been recognized as very important methods in circuit theory, communications and control science. Their sampled derivations have been encountered in different applications from science and engineering. In some situations the domain is restricted or other explicit conditions are imposed.

A critical issue is related to the singularities involved in the Hilbert transform computation, since we are confronted with an improper integral or with singularities. If the integral cannot be evaluated in a closed form (as it is the case with discrete input data), numerical implementation is in general complicated. Moreover, for a proper approach, localized errors in gain characteristic approach should lead to localized errors in phase characteristic approach.

Hilbert transform has the advantage of not requiring derivatives, but the serious disadvantage that it is not a bounded operator from L^∞ to L^∞. This situation can be controlled in the case of rational functions. On the other hand, to solve this problem, different approaches for gain-phase relationships in logarithmic frequency domain have been proposed. The conventional algorithms for recovering signals from parts of Fourier transform, involved applying the Hilbert transform on the Fourier transform log-magnitude or phase, to obtain the unknown component. An alternative approach consists in developing some iterative algorithms to reconstruct minimum (or maximum) phase signal from the corresponding Fourier transform magnitude or phase.

Overview of the Thesis

In this PhD thesis the aim is to study and to develop phase approximation methods from gain samples, for different applications in signals reconstruction. In general, our attention will be focused on those methods that required finally only a small number of gain samples to approximate the phase.

The first part of the study is comprised in *Chapter 2*; the most important theoretical aspects regarding the subject of the present thesis are presented: Hilbert transform, Kramers-Kronig relationships, Cauchy principal values, circuit functions and Bayard-Bode relationships.

In *Chapter 3* the useful test data for the different phase approximation methods are illustrated. The test functions used are: the Bode's circuit functions, the modified Bode's circuit functions, the lorentzian and the gaussian functions and the attenuation coefficient of a transmitted beam in a thick holographic grating, and also variants of this functions affected by perturbations, as well as test data obtained from autoregressive series.

Next chapters deal with more phase approximation methods: linear frequency domain approximation (Chapter 4) using compact and non-compact gain techniques; logarithmic frequency domain approximation (Chapter 5) using logarithmic derivative and difference derivative methods, Newton-Cotes and Simpson techniques; gaussian quadrature approach (Chapter 6).

In *Chapter 4* a new method for piecewise linear approximation of a function using a Divide-and-Conquer approach was developed. The algorithm of determining the slopes and the breakpoints for the piecewise linear approximation is illustrated and implemented. From the obtained results, one can conclude that the new piecewise linear approximation of a function method proposed is always convergent (in practical situations only a finite number of samples are available) and offers satisfactory results.

In *Chapter 5* logarithmic frequency domain phase approximation methods are presented: the method based on the logarithmic derivative, the method based on the logarithmic differences of order 1, 2 and 4, Newton-Cotes and Simpson techniques. The corresponding implementations algorithms were illustrated and also simulations using more data sets (also test data from ideal transfer function responses, as well as data affected by perturbations). The two approaches: Newton-Cotes and Simpson, can be successfully used as phase approximation methods in the logaritmic frequency domain.

In *Chapter 6* a phase approximation method, respectively a Hilbert (Kramers-Kronig) transform approximation based on gaussian quadrature approach is illustrated. Also test data from ideal transfer function responses, as well as data affected by perturbations were used. The proposed approach for the numerical evaluation of Hilbert transform, respectively for phase approximation give rather good results. The ease of implementation makes the proposed technique attractive as a means of singular integrals numerical evaluation point of view. all the intensive computation labor occurs in the determination of the weights and evaluation points, but this needs to be done only once.

Chapter 7 is dedicated to the phase approximation methods performances evaluation presented in this PhD thesis and also to the personal contributions illustration.

From the obtained results, one could say that:

- For classical RLC circuit functions (as are Bode's circuit functions) the most convenient phase approximations are given by gaussian quadrature and 4th order logarithmic difference approaches, and the most non-convenient approximations are given by non-compact gain technique;
- For circuit functions whose gain characteristics has zero slopes at zero and at very high frequencies (as are modified Bode's circuit functions), the use of non-compact gain technique or, sometimes, compact gain technique is recommended; if only a small numbers of samples are available for approximation, the use of 4th order logarithmic difference approach is recommended;
- For those functions that present only a principal lobe (as are lorentzian and gaussian functions), the evaluation of the unknown component is recommended to be done by the use of non-compact or compact gain support methods; if only a small numbers of samples are available for approximation, the use of Newton-Cotes or Simpson approaches are recommended;
- If the functions present a principal lobe and some secondary lobes the most convenient approximations of the phase are given by non-compact and compact support methods; if only a small numbers of samples are available for approximation, the use of trapezoidal or parabolic approaches are recommended.

Some the the obtained results are illustrated, in the case of L_1 norm, Figure 1, for noisy data.

Those methods that required a high number of gain samples or a high computational effort for a convenient phase approximation, were not the subject of this thesis:

- MacLaurin and trapezoidal formulas from linear gain sampling;
- Using of discrete Hilbert transform;
- Iterative evaluation methods (Gerchberg, Saxton, Fienup algorithms);
- Phase approximation using logarithmic gain derivatives series.

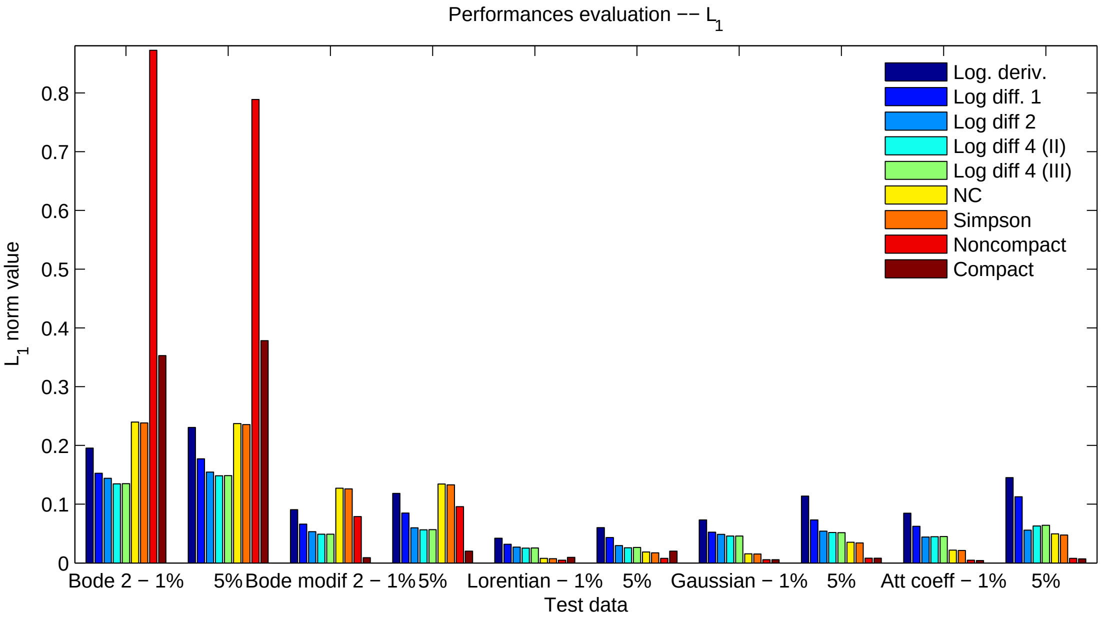

Figura 1: Performances evaluation – noisy data – L_1 norm.

Also, our study considered only the case of minimum phase functions; for non-minimum phase functions supplementary information is needed in order to obtain the phase approximation.

The implementations were done using *MATLAB 7.0.1* and *Mathematica 5.2*. Nevertheless, in this PhD thesis none *MATLAB* or *Mathematica* code was presented; we have been illustrated the algorithms, such that all the methods to can be implemented in every other software language, from case to case.

In general, increasing the number of points used for the approximation, the solutions accuracy increase. Also, increasing the frequency evaluation range, the approximations accuracy increase. From the obtained results, we can say that, for all test data sets used, an approximation in 30 points offers expected results, also for non-affected perturbation functions, as well as for functions affected by perturbations. the approximation in more than 30 points is not justified, at least in the studied cases; the computational evaluation increases by increasing the number of approximation points, but the error between the the exact function and its approximation does not decrease significant. However, we have experienced some problems in the approximation, when the function is very rapidly modifying in a relatively narrow frequency range, but using 30 points the phase approximation behaves well, even if the phase is very rapidly modifying.

Author's Contributions

The *personal contributions* are to be found mostly in Chapters 3-6 and in Section 7.1. New methods for phase approximations from gain samples based on quadrature relationships are developed, and the existing methods are improved regarding their behavior in the case of data affected by perturbations. The major contributions brought by this PhD thesis are:

- For testing the phase approximation methods from gain samples there have been used a vast collection of data sets, which involve all the type of experimental data previously used by different authors, at which new test data are added;
- All the algorithms are described in pseudocode;
- A new method for piecewise linear approximation of a function using a Divide-and-Conquer technique was developed;
- For the linear frequency domain, the compact and non-compact gain techniques were implemented and compared;
- The method based on the logarithmic derivative, the method based on the logarithmic differences of order 1, 2 and 4, Newton-Cotes and Simpson techniques were implemented and compared for the logarithmic frequency domain;
- A phase approximation method, respectively a Hilbert (Kramers-Kronig) transform approximation based on gaussian quadrature approach was implemented;
- For the phase approximation methods analyzed in this thesis, a performance evaluation have been done, from which the implementation methods function of gain samples disposal and behavior can be chosen.