P1: An analog signal

$$x_a(t) = \cos(200\pi t) - 3.5\cos(600\pi t)$$

is sampled by 0.5 kHz. Evaluate the DFT coefficients, for $N = 20$.

P2: An analog signal $x_a(t)$ is sampled by $T = 0.01$ s. The resulting sequence is:

n	0	1	2	3	4	5
$x(n)$	5	-1.5	6.5	-3	6.5	-1.5

Evaluate the spectrum using the 6-point DFT and specify to which analog frequency corresponds each $X(k)$.

P3: Consider the analog signal

$$x_a(t) = A\cos(200\pi t) - B\cos(800\pi t)$$

sampled by 1 kHz.

a) Determine the period of the obtained discrete-time signal.
b) Evaluate the 20-point DFT of the sequence.
c) Repeat the previous parts for a sampling frequency of 0.5 kHz. Explain the obtained result.

P4: A speech signal is sampled at a rate of 20,000 samples/sec. A segment of length 1024 samples is selected and its DFT is computed.

a) What is the time duration of the segment of speech?
b) What is the frequency resolution between DFT values?
c) What happens if only the even samples are retained?

P5: Evaluate the 8-point DFT of the sequence $x(n) = \{\underset{\uparrow}{1}, 1, 1, 1, 1, 1, 0, 0\}$. Sketch the magnitude and the phase spectra of the obtained DFT coefficients.

P6: Evaluate the 8-point DFT for the next sequences:

$$x(n) = \cos\frac{\pi}{2}n, \quad n = \overline{0,7};$$

$$h(n) = 2^n, \quad n = \overline{0,7}.$$

P7: Evaluate the linear and the circular convolution ($N = 4$ and $N = 8$) of the sequences:

$$x_1(n) = \{\underset{\uparrow}{2}, 2, 1, 1\};$$

$$x_2(n) = \{1, \underset{\uparrow}{2}, 3, 4\}.$$

P8: Evaluate the linear and the circular convolution ($N = 8$) of the sequences:

$$x_1(n) = \{\underset{\uparrow}{1}, 2, 3, 4, 5\};$$

$$x_2(n) = \{\underset{\uparrow}{1}, -1, 2, -2, 0\}.$$

P9: Evaluate the power spectra for the periodic sequence:

$$x(n)=\{\underset{\uparrow}{2},2,2,2,0,0,0,0\}, \qquad x(n)=x(n+8)\ .$$

P10: A linear time-invariant system is described by the constant coefficient difference equation:

$$y(n)-\frac{3}{4}y(n-1)+\frac{1}{8}y(n-2)=2x(n-1)\ .$$

a) Determine the homogenous solution.
b) Determine the solution, if:

$$y(-1)=1, \qquad y(0)=0 \quad \text{and} \quad x(n)=\cos\frac{2\pi n}{5}u(n)\ .$$

P11: A linear time-invariant system is described by the constant coefficient difference equation:

$$y(n)=0.7y(n-1)-0.06y(n-2)+2x(n)-x(n-2)\ .$$

a) Determine the unit impulse response.
b) Determine the unit step response, in zero initial conditions.
c) Evaluate the stability of the system.

P12: A linear time-invariant system is described by the constant coefficient difference equation:

$$y(n)-5y(n-1)+6y(n-2)=3x(n-1)\ .$$

a) Determine the homogenous solution.
b) Determine the unit impulse response.
c) Determine the unit step response.

P13: Evaluate, function of $\cos\omega$, the squared magnitude of the frequency response function, for the system described by the input-output relationship:

$$y(n)=-0.1y(n-1)+0.2y(n-2)+x(n)+x(n-1)$$

P14: Knowing that:

$$y(-1)=y(-2)=1,$$

determine the unit step response for the system described by:

$$y(n)=0.9y(n-1)-0.81y(n-2)+x(n)\ .$$

P15: A linear time-invariant system is described by the constant coefficient difference equation:

$$y(n)=0.7y(n-1)-0.1y(n-2)+2x(n)-x(n-2)\ .$$

a) Determine the unit impulse response.
b) Determine the unit step response.
c) Evaluate the stability of the system.

P16: Derive the radix-2 decimation-in-time FFT algorithm ($N = 16$).

P17: Determine all possible discrete time signals associated with the z-transform:

$$X(z) = \frac{5z^{-1}}{\left(1 - \frac{1}{2}z^{-1}\right)\left(3 - z^{-1}\right)}.$$

For each obtained sequence specify the corresponding region of convergence.

P18:

a) Evaluate and sketch the frequency response for:

$$y(n) - \frac{1}{2}y(n-1) = x(n) + 2x(n-1) + x(n-2).$$

b) Write the constant coefficient difference equation corresponding to the system described by the frequency response function:

$$H(\omega) = \frac{1 - \frac{1}{2}e^{-j\omega} + e^{-j3\omega}}{1 + \frac{1}{2}e^{-j\omega} + \frac{3}{4}e^{-j3\omega}}.$$

P19: Consider the LTI system described by:

$$y(n) = x(n) - x(n-4).$$

a) Sketch the magnitude and the phase of the frequency response function.

b) Evaluate the output of the system to the input signal:

$$x(n) = \cos\frac{\pi}{2}n + \cos\frac{\pi}{4}n, \qquad -\infty < n < \infty.$$

P20: Evaluate the unit impulse response and the frequency response, and sketch them, for the system:

$$y(n) = \frac{1}{2}y(n-1) + x(n) + \frac{1}{2}x(n-1), \qquad n \in \mathbb{Z}.$$

P21: An LTI system is described by the difference equation:

$$y(n) = x(n) + x(n-10).$$

a) Sketch the magnitude and the phase of the frequency response function.

b) Evaluate the output of the system to the input signal:

$$x(n) = \cos\frac{\pi n}{10} + 3\sin\left(\frac{\pi n}{3} + \frac{\pi}{10}\right), \qquad -\infty < n < \infty$$

P22: An LTIS is described by:

$$y(n)=x(n)+5x(n-10).$$

a) Evaluate the magnitude and the phase of the frequency response function.
b) Evaluate the output of the system to the input signal:

$$x(n)=5+6\cos\left(\frac{2\pi n}{5}+\frac{\pi}{2}\right),\qquad -\infty<n<\infty.$$

P23: Evaluate and sketch the frequency response characteristics (magnitude, phase and group delay) of a moving average (MA) filter, for $M=6$, with equal weights.

P24: Design a two-pole band-pass filter, with pass-band centered on $\omega=\pi/2$ and zeros in $\omega=0$ and $\omega=\pi$. The magnitude of the frequency response function is $1/\sqrt{2}$ at $\omega=4\pi/9$ ($\cos\frac{8\pi}{9}\cong-0.94$).

P25: Design a linear-phase FIR filter, of fourth order, for which:

$$H_r(0)=1 \text{ şi } H_r\left(\frac{\pi}{3}\right)=\frac{1}{2}.$$

P26: Design an FIR filter which completely rejects the frequency $\omega_0=\pi/4$. Evaluate the output of the filter to the excitation:

$$x(n)=\sin\frac{\pi n}{4}u(n),\qquad n=\overline{0,4}$$

P27: Design an FIR filter in order to reject the normalized frequencies $\frac{1}{10}$, $\frac{1}{20}$, $\frac{1}{30}$. The filter gain is 8 at $\omega=0$. Write the transfer function as a product of second order subsystems with real-valued coefficients.

P28: Evaluate the magnitude and the phase of the frequency response function, for the system characterized by the transfer function:

$$H(z)=1+z^{-1}+\cdots+z^{-16}.$$

For a sampling frequency of 1 kHz, determine the frequencies of the sinusoids that will be rejected by the filter.

P29: Sketch the direct forms and the lattice/lattice-ladder structure, for the LTI systems:

a) $2y(n)+y(n-1)-4y(n-3)=x(n)+3x(n-5)$;
b) $y(n)=x(n)-x(n-1)+2x(n-2)-3x(n-4)$.

P30: Sketch the lattice structure for the linear-phase FIR filter of length 4, for which:

$$H_r(0)=1 \text{ şi } H_r\left(\frac{\pi}{2}\right)=\frac{1}{2}.$$

P31: Sketch the lattice-ladder structure of the system:

$$H(z) = \frac{z^3 + z}{(z+0.5)(z^2 + z + 0.5)}.$$

P32: Sketch the parallel and the cascade structures for the system:

$$H(z) = \frac{1 + z^{-1}}{\left(1 - \frac{1}{2}z^{-1}\right)\left(1 - 0.9e^{j\frac{\pi}{3}}z^{-1}\right)\left(1 - 0.9e^{-j\frac{\pi}{3}}z^{-1}\right)}$$

P33: Sketch the parallel and the cascade structures for the system:

$$H(z) = \frac{10\left(1 - \frac{1}{2}z^{-1}\right)\left(1 - \frac{2}{3}z^{-1}\right)\left(1 + 2z^{-1}\right)}{\left(1 - \frac{3}{4}z^{-1}\right)\left(1 - \frac{1}{8}z^{-1}\right)\left[1 - \left(\frac{1}{2} + j\frac{1}{2}\right)z^{-1}\right]\left[1 - \left(\frac{1}{2} - j\frac{1}{2}\right)z^{-1}\right]}$$

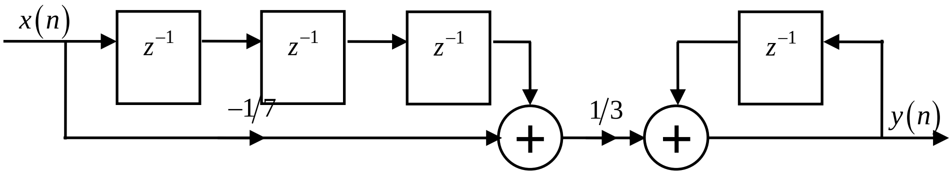
P34: Consider the system described by the transfer function:

$$H(z) = \frac{1 + \frac{1}{5}z^{-1}}{\left(1 - \frac{1}{2}z^{-1} + \frac{1}{3}z^{-2}\right)\left(1 + \frac{1}{4}z^{-1}\right)},$$

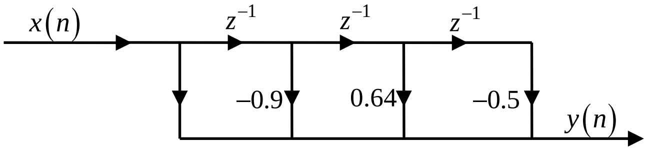
Sketch:

a) The direct forms.
b) The cascade structure.
c) The parallel structure.

P35: Evaluate and sketch the unit impulse response and compute the magnitude of the frequency response function for the system described by the block diagram:



P36: Realize the lattice structure for the FIR described by the block diagram:



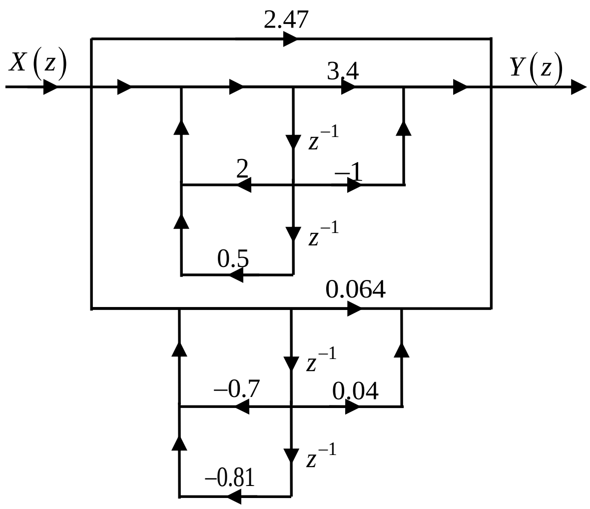
P37: Realize the lattice structure and evaluate the response to the unit ramp sequence, in zero initial conditions, for the system described by the transfer function:

$$H(z) = \frac{1}{1 + 2z^{-1} + \frac{1}{3}z^{-2}}$$

P38: Determine the unit impulse response of the FIR filter with lattice parameters $k_1 = 0.6$, $k_2 = 0.3$, $k_3 = 0.5$ and $k_4 = 0.9$. Evaluate the frequency response function.

P39: Determine all possible FIR filters with lattice parameters $k_1 = 1/2$, $k_2 = 0.6$, $k_3 = -0.7$ and $k_4 = 1/3$. Compute their responses to the unit step sequence.

P40: Sketch the lattice-ladder structure for the system described by the block diagram:



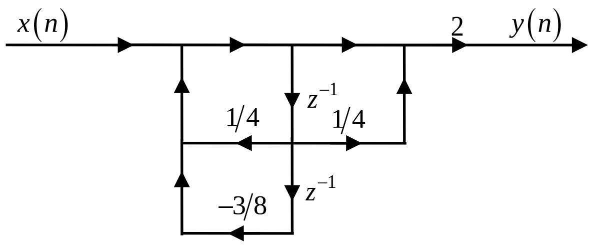
P41: Evaluate the magnitude, the phase of the frequency response function and the group delay, for the linear time-invariant system:

$$y(n) = 1.8y(n-1) - 0.81y(n-2) + x(n) + 0.95x(n-1)$$

P42: Consider the FIR filter with the lattice coefficients $k_1 = 0.65$, $k_2 = -0.34$ and $k_3 = 0.8$.

a) Evaluate the impulse response.
b) Sketch the direct form.

P43: Consider the system described by the block diagram:



a) Evaluate the transfer function.
b) Determine the impulse response.
c) Write the input-output relationship.

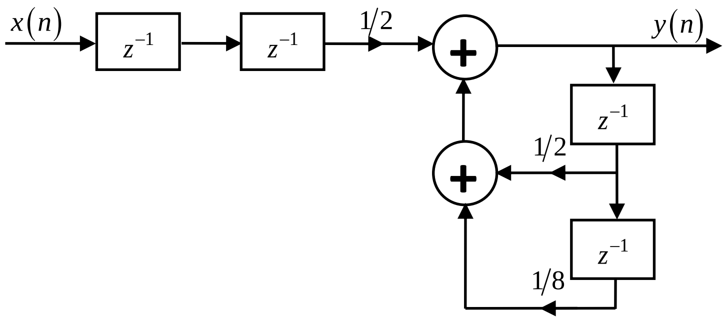
P44: Realize the lattice structure and evaluate the response to the unit ramp sequence, in zero initial conditions, for the system described by the transfer function:

$$H(z) = 1 + 2z^{-1} + \frac{1}{3}z^{-2}.$$

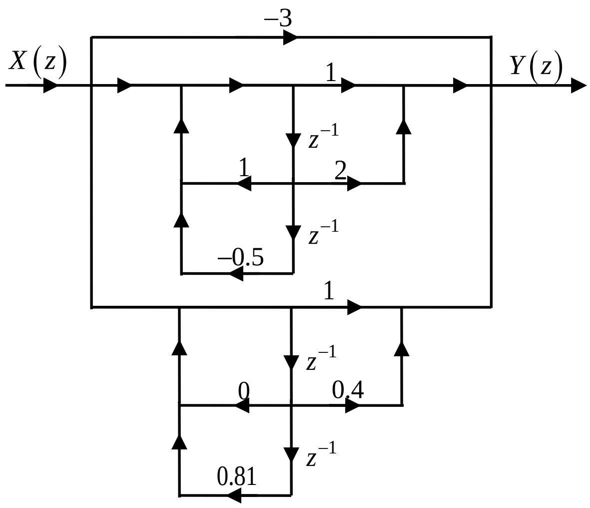
P45: Sketch the parallel and the cascade structures for the system:

$$H(z) = \frac{3+5z^{-1}-2z^{-2}}{\left(1-\frac{1}{2}z^{-1}\right)\left(1-3z^{-1}\right)\left(1-\frac{1}{4}z^{-1}\right)}.$$

P46: Evaluate the unit impulse response and the frequency characteristics (magnitude and phase) for the system described by the block diagram:



P47: Consider the system described by the block diagram:



Evaluate:

a) Evaluate the transfer function.
b) Evaluate the frequency response function.
c) Determine the impulse response.
d) Write the input-output relationship.

P48: Convert the analog filter with system function

$$H_a(s) = \frac{s+0.1}{(s+0.1)^2+9}$$

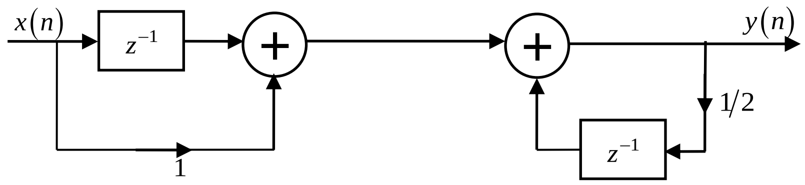
into a digital IIR filter by means of the impulse invariance method ($T = 0.1$ s and $T = 0.5$ s). For which value of T, the alias effect is more prevalent?

P49: Compute the Fourier transform for the following sequences:

a) $x(n) = \begin{cases} 1, & n = \overline{0, M} \\ 0, & \text{otherwise} \end{cases}$

b) $x(n) = \begin{cases} \frac{1}{2}\left(1 + \cos\frac{2\pi n}{M}\right), & n = \overline{0, M} \\ 0, & \text{otherwise} \end{cases}$

P50: Evaluate the unit impulse response of the system:



Proof that:

$$h(n) = [\delta(n) + \delta(n-1)] * \left[\left(\frac{1}{2}\right)^n u(n)\right]$$

where '$*$' stands for 'linear convolution'.