To solve for b_0 and b_1 we use:

$$h(n) = -a_1 h(n-1) - a_2 h(n-2) - \ldots - a_N h(n-N) + b_n,$$

with $h(n) = h_d(n)$. For $n = 0$ we obtain b_0:

$$1 = \frac{1}{3} \cdot 3 \cdot 3 + b_0 \quad \Rightarrow b_0 = 2.$$

For $n = 1$ we obtain b_1:

$$3 \cdot \frac{1}{3} = \frac{1}{3} \cdot 3 + b_1 \quad \Rightarrow b_1 = 0.$$

Thus $H(z) = H_d(z)$. This example illustrates that the Padé approximation results in a perfect match to $H_d(z)$ when the desired system function is rational and we have prior knowledge of the number of poles and zeros in the system.

However, in general, this is not the case in practice, since $h_d(n)$ is determined from some desired frequency response specifications $H_d(\omega)$. In such a case Padé approximation may not result in a good filter design. An effective approach in using the Padé approximation is to try different values of M and N until the frequency responses of the resulting filters converge to the desired frequency response within some small, acceptable approximation error.

10.4 Exercises

1. Consider a Cebyshev type II digital HPF, with 4 poles and 4 zeros, with the system function:

   $$H_d(z) = \frac{0.076945 - 0.19009z^{-1} + 0.25374z^{-2} - 0.19009z^{-3} + 0.076945z^{-4}}{1 + 0.80034z^{-1} + 0.73056z^{-2} + 0.17774z^{-3} + 0.035329z^{-4}}$$

   Using the Padé approximation method and considering the impulse response length equal by 50, compare the method's performances for: $M = \{2; 4; 6\}$ and $N = \{2; 4; 6\}$.

2. Consider the filter given in exercise 1; approximate it using the inverse filter (the least squares design method).

3. Consider the filter given in exercise 1; approximate it using Prony's design method.

4. Using Yule-Walker method, synthesize a BRF of order 8, with the stopband between 0.3 and 0.6 and the cutoff passband frequencies 0.25 and

0.65.

5. Consider an LTIS described by the transfer function:

$$H(z) = \frac{0.05634(1+z^{-1})(1-1.01666z^{-1}+z^{-2})}{(1-0.68z^{-1})(1-1.4461z^{-1}+0.7957z^{-2})}.$$

Sketch the pole-zero diagram and the frequency response characteristics (the magnitude, the phase and the group delay).