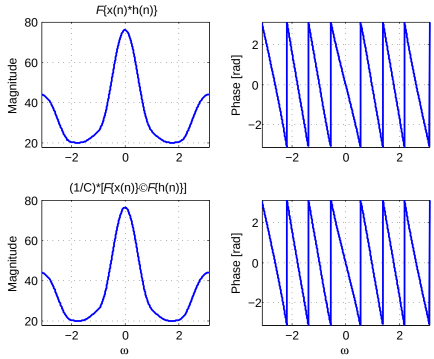


**Figure 6.6:** Fourier transform of the product sequence, and circular convolution of the individual Fourier transforms

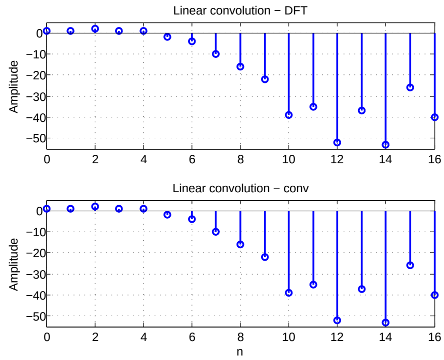


**Figure 6.7:** Linear convolution between two sequences evaluated directly and by use of DFT

The considered sequences for the MATLAB script `L6_5.m` are the excitation:

$$x(n) \quad = \quad \{ \underset{\uparrow}{1}, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

and the impulse response:

$$h(n) \quad = \quad \{ \underset{\uparrow}{1}, -1, 1, -2, 1, -3, 1, -4\}.$$

In Fig. 6.7 is graphed the result obtained by evaluating the linear convolution, and also the one obtained by evaluating the circular convolution.

```
% L6_4 - linear convolution evaluated using the circular convolution
x = [1 2 3 4 5 6 7 8 9 10]; h = [1 -1 1 -2 1 -3 1 -4];
N=length(x); M=length(h); n=0:N+M-2; y_lin=conv(x, h);
xe = [x zeros(1, M-1)]; he = [h zeros(1, N-1)];
X=fft(xe); H=fft(he); Y=X.*H;  y_dft=real(ifft(Y));
subplot(211); stem(n, y_dft); title('Linear convolution-DFT')
subplot(212); stem(n, y_lin); ylabel('Amplitude');
xlabel('n'); title('Linear convolution - conv');
```

## 6.4 Exercises

1. Determine the linear convolution of the sequences:

$$x_1(n) = 1.5\cos\left(2\pi 0.1n + \frac{\pi}{4}\right) \quad \text{and} \quad x_2(n) = |10 - n|, \; n = \overline{0, 20}.$$

Plot the two sequences and the result of the linear convolution. Which is the length of the linear convolution sequence?

2. The impulse response of an LTIS is:

$$h(n) \quad = \quad \begin{cases} \exp(-0.1n), & n = \overline{0, 31}, \\ 0, & \text{otherwise.} \end{cases}$$

At the input of the system the sequence $x(n) = u(n) - u(n-20)$ is applied. Determine the output of the system using the linear convolution.

3. The impulse response of an LTIS is:

$$h(n) = \begin{cases} \exp(-0.15n), & n = \overline{0,31}, \\ 0, & \text{otherwise}. \end{cases}$$

At the input of the system the sequence $x(n) = u(n) - u(n-30)$ is applied. Determine the output of the system using the linear convolution.

4. Consider the system: $H(z) = \dfrac{z}{z-0.5}$. Evaluate:

- The unit step response;
- The unit ramp response;
- The response to the sequence $x(n) = 10\cos\dfrac{\pi n}{3} \cdot u(n)$;
- The response to the sequence $x(n) = 10 \cdot 0.5^n \cdot u(n)$.

5. Consider the system described by the $z$-domain transfer function:

$$H(z) = \frac{z-1}{(z-0.25)(z-0.5)}.$$

- Determine the first 100 samples of the unit step response sequence;
- Express the system function as: $H(z) = H_1(z) + H_2(z)$; determine the unit step response of the individual blocks and then add the results. Compare the obtained result with the result obtained in the first part.

6. Two linear systems are connected in cascade:

$$h_1(n) = \{\underset{\uparrow}{2}, 3, 2, 1, -0.5, 1, 2, 4\} \quad \text{and} \quad h_2(n) = \{\underset{\uparrow}{3}, -1, 5, 0, 2, 6\}.$$

- Generate an arbitrary input sequence $x(n)$ (i.e., a sinusoidal sequence); evaluate the output sequence of the first system, using the linear convolution, and then evaluate the output sequence of the cascade-formed by the two systems;
- If the cascade order is changed, repeat the operations involved in the precedent part. What can you conclude?
- Suppose that the second system is characterized by the input-output relation: $y(n) = 0.01[x(n)]^2$, and the first system remains unchanged. Repeat the precedent parts and compare the output resultant sequences.

7. Evaluate the circular convolution of the sequences:

$$x_1(n) = 1.1\cos\left(\pi 0.25n + \frac{\pi}{6}\right) \quad \text{and} \quad x_2(n) = (-2)^n,\ n = \overline{0,10}.$$

Plot the two sequences and the sequence obtained after the evaluation of the circular convolution. Which is the length of the circular convolution?

8. Consider the sequences:

$$x_1(n) = 1.1\sin\left(2\pi 0.05n + \frac{\pi}{4}\right) \quad \text{and} \quad x_2(n) = (-1)^n,\ n = \overline{0,15}.$$

Write a MATLAB script in order to evaluate:

- The linear convolution;
- The 16-point circular convolution in two ways (using `circonv` and `fft`);
- The circular convolution in the minimum number of points required in order to obtain the same result as in the case of the linear convolution, in two ways (using `circonv` and `fft`).

9. Evaluate the linear and the circular (using minimum length DFT required) convolution of the sequences: $x_1(n) = u(n) - u(n-20),\ n = \overline{0,30}$ and $x_2(n) = (-0.7)^n,\ n = \overline{0,20}$. Which is the minimum $N$ such that the values of the two convolutions to be the same? Graph the two sequences and also those obtained after the evaluation of linear, and circular convolution, respectively.

10. Consider the sequences:

$$x_1(n) = \{\underset{\uparrow}{3}, 4.2, 11, 0, 7, -1, 0, 2\}, \qquad x_2(n) = \{\underset{\uparrow}{1.2}, 3, 0, -0.5, 2\},$$

- Evaluate the linear convolution (using `conv`) between $x_1(n)$ and $x_2(n)$. Which is the length of the result?
- In some cases, it should be convenient to evaluate the convolution using the Fourier transform. At the beginning, evaluate the linear convolution in a some way inconsistent manner. Extend $x_2(n)$ with three zeros, so that both sequences to have the same length. Evaluate then the 8-point DFT for the two sequences. After multiplying the two DFTs, evaluate the IDFT of the product $X_1(k)X_2(k)$. In what measure the result is identical with the one obtained through the linear convolution. How many samples are accurate? Why?
- Which is the minimum length DTF that must be used, so that through the preceding procedure to obtain the same result as in the case of linear convolution? Pad both sequences by zeros, till both of them are of length equal by the minimum required to evaluate accurate the linear convolution using the DFT. Repeat the anterior part.
- Pad with five zeros the two sequences, so that their length is greater

than the minimum required. Repeat the anterior part and specify to what extent a greater number of samples affect the result.

11. Consider the sequence:

$$x(n) \quad = \quad \{ \underset{\uparrow}{3}, 2, 7, 1, 4\}.$$

- Evaluate the 5-point DFT for the sequence $x(n)$. Multiply the DFT by a complex exponential: $e^{-j\frac{2\pi k}{5}}$. Compute the IDFT of the product, that is to find the sequence: $x_1(n) = \text{IDFT}\{X(k)e^{-j\frac{2\pi k}{5}}\}$. Take into account only the real part of the sequence $x_1(n)$, the imaginary part being the result of the roundoff errors. Compare $x_1(n)$ by $x(n)$. Are these sequences obtained by circular translation?
- Repeat the anterior part to obtain a circular shift by three samples.
- How can you modify this technique in order to evaluate the linear convolution?