$$\begin{aligned}
\Rightarrow h(n) &= [0.275(-j0.67)^n + 0.275(j0.67)^n]u(n) \\
&= 0.275(0.67)^n\left(e^{-j\frac{\pi n}{2}} + e^{j\frac{\pi n}{2}}\right)u(n) \\
&= 0.55 \cdot 0.67^n \cdot \cos\frac{\pi n}{2} \cdot u(n).
\end{aligned}$$

8.4 Exercises

1. For the filters designed at exercise 1 represent the frequency response characteristics (phase and group delay). Explain the symmetry of LP characteristics versus HP. For each filter write the system function such that the maximum value of the frequency response function to be 1 (0 dB).

2. Plot the magnitude and the phase of the frequency response function for two notch digital filters. The notch filter must eliminate the frequency $\omega = \dfrac{\pi}{3}$. For the poles consider $r = 0.6$ and $r = 0.96$, respectively.

3. Design an IIR LPF, of first-order, with the cutoff frequency $\omega_c = 0.3\pi$ and unitary gain in the passband. Plot the frequency response characteristics (magnitude, phase and group delay).

4. Consider the FIR filter described by the input-output relationship:
$$y(n) = \frac{1}{4}\left[x(n) + x(n-1) + x(n-2) + x(n-3)\right].$$
Evaluate and sketch the impulse response and the frequency characteristics.

5. An LTIS is described by the transfer function: $H(z) = \dfrac{z}{z-0.9}$.
   - Evaluate and sketch the impulse response;
   - Evaluate and sketch the frequency response characteristics;
   - Evaluate the output of this filter to the excitation $x(n) = \sin(2\pi 0.05n)$, for $n = \overline{0, 499}$. Compare the excitation with the output sequence. How are affected the amplitude and the phase of the input sinusoid?
   - Repeat the previous part for $x(n) = \sin(2\pi 0.1n)$, $n = \overline{0, 499}$.

6. Two continuous time signals are considered $x_a(t)$ and $y_a(t)$, which are in an integral relationship: $y_a(t) = \int_0^t x_a(t)dt$. The integral can be approxi-

mated using the trapezoidal rule as follows:

$$y_a(t) \simeq y_a(t_0) + \frac{t - t_0}{2}[x_a(t) + x_a(t_0)].$$

A discrete integrator can be represented by the finite difference equation:

$$y(n) = y(n-1) + \frac{T}{2}[x(n) + x(n-1)],$$

where $x(n)$ and $y(n)$ represent the sampled signals derived from $x_a(t)$ and $y_a(t)$.

- Determine the transfer function $H(z)$ of the discrete integrator;
- Generate two vectors to describe the discrete integrator. Chose $T = 0.1$ s;
- Consider the signal: $x_a(t) = 0.9^t \sin(2t)$. Its integral can be approximated by the discrete integrator. For this purpose, this signal is sampled by $T = 0.1$ s and it is passed through the integrator. Evaluate the first 100 samples for the output sequence;
- Repeat previous parts for $T = 0.05$ s.

7. Consider the LTIS described by the system function: $H(z) = \dfrac{1}{1 - z^{-N}}$.
   - Create a variable to describe this system, and then generate 100 samples of the system's impulse response ($N = 10$);
   - Evaluate and sketch the frequency response characteristics;
   - Generate the sequence $x(n) = 9 - n$, for $n = \overline{0, 9}$. Pad $x(n)$ by 90 zeros and then pass this new sequence through the filter. Evaluate the first 100 samples of the response sequence.